AN ECONOMICAL DESIGN OF REINFORCED CONCRETE FRAME STRUCTURE

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ABSTRACT. In today's world of baffling prices of construction materials, the optimization is the only tool to cut down the structural cost. The present study has considered one of the important optimization techniques – PSOGSA – a hybrid of particle swarm optimization and gravitational search algorithm - to emphasize its role in structural optimization. The objective of this study is to explore combined advantages of different heterogeneous algorithms. Since a structure is a combination of different structural elements, it can be optimized either as a single entity or by partitioning it into sub-structures, wherein different elements are considered separately. In the present case, a frame structure in which beams and columns have been optimized as discretised element to highlight the effect of optimization technique. The technique has been found easy to understand and implement with encouraging results. The total cost of a RC frame includes the cost of both concrete as well as steel. The entire formulation for optimal cost design of frame includes the cost of beams and columns. An example has been considered to emphasis the validity of this optimum design procedure and results have been compared with earlier study.

Keywords: Structural optimization, Reinforced concrete frame, Particle swarm optimization, Gravitational search algorithm, Hybrid technique.

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INTRODUCTION

Recent development in reinforced concrete (RC) construction has taken giant leaps across many countries. The popularity of RC construction owes to the lack of robust steel industry in different countries, which makes high rise steel construction very expensive. RC frames are the most preferred structural system for exhibiting excellent performance under vertical and gravity loads. These are the intricate part of a multi-storeyed and high rise building system.

Structural optimization is one of the important decision making problem broadly relates to designing a structure at minimum cost, while simultaneously fulfilling all design requirements. It has become the domain of several mathematicians and structural designers and gained their interest to develop efficient procedures to enhance the structural performance. Many classical and heuristic optimization techniques are available for the use of structural design applications. Classical approach of optimization normally referred to gradient based search approach and the heuristic technique umbrella includes different search algorithms like evolutionary algorithm (EA), genetic algorithm (GA), simulated annealing (SA), ant colony optimization (ACO), harmony search (HS), particle swarm optimization (PSO) and other hybrid algorithms etc. Although heuristic techniques are made up of simple algorithms but they require great computational efforts and turned up to be efficient in performance in structural optimization domain.

In the available literature of optimization of RC structures, it is observed that optimality criterion (OC) method [1-2] and genetic algorithms [3-9] are popular choice of many researchers for optimizing complex non linear problems of RC structural design. Particle swarm optimization [10-11] has also been widely used for optimum design of steel structures due to its simplicity and fast convergence but its application for optimum design of RC structures has been found limited. PSO possesses the major difficulty in making right balance between global investigation of the search space and refined search around local optima [12-14]. To improve upon this specific problem, PSO is hybridized with many other approaches such as ACO and HS by the researchers [15-16].

In present study, PSO is hybridized with gravitational search algorithm (GSA) to develop hybrid PSOGSA. The basic idea of combining standard PSO with GSA is suggested by *Mirjalili and Hashim* [17]. GSA has laden with good local search capacity [18]. They combined social thinking ability of PSO and local search capability of GSA to propose hybrid PSOGSA. The application of PSOGSA technique on optimizing RC frame elements is an unfold effort which is performed in this study and results are compared with previous works to test the efficiency and effectiveness of PSOGSA.

METHODOLOGY

The computer aided analysis and optimum design procedure for plane reinforced concrete frame subjected to gravity and lateral loads has been attempted in this study. For this purpose, frame was discretised into beams and columns. The design and optimization of beams and columns have been done separately. The limit state method based on IS456:2000 [19] was adopted for design of different elements of the frame. The total cost of RC frame constitutes the cost of beams and columns and was considered as objective function.

$$Z_{Total\ cost} = \sum_{n=1}^{NB} Z_{Beam} + \sum_{n=1}^{NC} Z_{Column} \tag{1}$$

NB = No. of beams in a frame; NC = No. of columns in a frame

The cost of reinforced concrete structural element (beam or column) primarily includes cost of concrete, steel and formwork and has been calculated as:

$$C = C_{st}V_{st} + C_C V_C + C_f A_f \quad (2)$$

C is the total cost of structural element; C_{st} cost of steel per unit volume of steel; V_{st} total volume of steel ; C_C cost of concrete per unit volume of concrete; C_f cost of a unit area of formwork per unit volume of concrete; V_C total volume of concrete. A_f total area of formwork. Dividing equation (2) by C_C as follows,

$$\frac{C}{C_C} = \frac{C_{st}}{C_C} V_{st} + V_C + \frac{C_f}{C_C} A_f \quad (3)$$

Substituting $\frac{C}{C_C} = Z$ (Objective function), $\frac{C_{st}}{C_c} = \alpha$ (Cost ratio of steel to concrete), $\frac{C_f}{C_c} = \alpha_1$ (cost ratio of formwork to concrete)

$$Z = \alpha V_{st} + V_C + \alpha_1 A_f \quad (4)$$

$V_C = V_G - V_{st}$ in the equation (4), it becomes

$$Z = (\alpha - 1)V_{st} + V_G + \alpha_1 A_f \quad (5)$$

Since C_C is a constant parameter for a given place, the objective function Z represents total cost of the frame that shall be minimized. Volume of steel (V_{st}) depends upon area of steel and its provided length. Similarly, gross volume of the element (V_G) depends upon its cross sectional area and length.

Z remained similar for different structural elements of the given frame but the constraints varied from one structural element to another.

Constraints for beam design and its optimization

Moment capacity consideration

For a given beam, the cross-sectional dimensions (depth and width) and area of steel to be provided at the ends and at bottom shall be such that its design moment of resistance is greater than actual moments to be borne by it at the respective sections.

Deflection consideration

For spans up to 10 m, the vertical deflection of a continuous beam shall be considered within limits if the ratio of its span (l) to its effective depth is less than 26. For spans above 10 m, factor 26 is multiplied by $\frac{10}{l}$.

Minimum width of beam

From practical consideration, the beam shall be wide enough to accommodate at least two bars of tensile steel of given diameter. Minimum width has been kept as user's input parameter.

Slenderness limit of beam from lateral stability consideration

As per IS 456: 2000, a continuous beam shall be so proportioned that the clear distance between lateral restraints does not exceed $60b_B$ or $250\ b_B^2/d_B$, whichever is less.

b_B = width of beam; d_B= effective depth of beam

Depth of neutral axis

To ensure that tensile steel does not reach its yield stress before concrete fails in compression so as to avoid brittle failure, the maximum depth of neutral axis has been restrained.

Minimum and maximum reinforcement steel

The minimum and maximum area of tensile steel (*Ast)* to be provided shall be taken as mentioned in relevant code of practice (IS456:2000).

Shear capacity consideration

The nominal shear stress in concrete should not exceed the maximum shear stress 0.6375 $\sqrt{fck}$ *N/mm*^{2}.

fck =Characteristic compressive strength of concrete in N/mm^2

Constraints for column design and its optimization

Axial load capacity of column

The axial load carrying capacity of the column shall be greater than the load to be borne by it.

Moment capacity of column

The moment carrying capacity of the column shall be greater than the moment to be borne by it.

Longitudinal reinforcement in column

The cross-sectional area of longitudinal reinforcement shall vary between 0.8 to 4 percent of the gross cross-sectional area of the column (although the Indian code denotes higher limit to be 6 percent, but due to practical difficulties in placing and compacting of concrete at places where bars are to be lapped, a lower percentage has been recommended).

Minimum number of longitudinal rebars

The number of longitudinal bars provided in a column shall not be less than 4.

Maximum peripheral distance between longitudinal rebars

The spacing of longitudinal bars measured along the periphery of column shall not be more than 300 mm.

Cross-section of the column

From practical point of view, the width of column shall be equal to or greater than the width of beams coming on it and also its cross-sectional dimensions shall be in sync with the size of the column lying immediately beneath it.

Design variables

In this study, for beam optimization depth d_B and width b_B of beam section were considered as independent design variables. Other variables like area of steel at the ends (Ast_{end}) and in the middle (Ast_{mid}) were derived from these two independent design variables. Similarly for column optimization, percentage area of longitudinal reinforcement (*p*) and ratio of depth of neutral axis to overall depth of column (*k)* were selected as independent design variables and rest of variables like cross sectional dimensions were obtained from these two values.

OVERVIEW OF HYBRID PSOGSA

Determination of global optimal solution among all possible inputs is the aim of implementing any optimization algorithm and to improve the performance, hybridization of two or more algorithms is performed. Several heuristic algorithms have been combined to form hybrid methods for optimization problems. The basic idea of combining Standard PSO with GSA was suggested by Mirjalili and Hashim (2010) [17]. They combined social thinking ability of PSO and search capability of GSA.
In order to explain this algorithm, a system with N masses (agents) is considered in which the position of the i^{th} mass is defined as:

$$X_i = (x_i^1, .. x_i^2 \dots, x_i^n \dots \dots .. x_i^d) , i = 1,2,3 \dots . N \qquad (6)$$

x_i^d is the position of i^{th} mass in the d^{th} dimension, and n is the dimension of the search space. In this case, the positions of masses are the candidate solutions for the problem, which at the next iterations of the algorithm will be improved. According to Rashedi (2009) [18], each agent’s mass is calculated after the evaluation of the current population’s fitness and considered as a candidate solution. After initialization, gravitational force, gravitational constant, and resultant forces among agents are calculated. After calculating the accelerations and with updating the best solution so far, the velocities of all agents can be calculated using (14). Finally, the positions of agents are defined as (15). The process of updating velocities and positions will be stopped by meeting an end criterion.

$$m_i\ (\mathrm{t}) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \tag{7}$$

$$M_i(\mathrm{t}) = \frac{m_i(\mathrm{t})}{\sum_{j=1}^{N} m_i(\mathrm{t})}$$

For any minimization problem

$$best(t) = \ min_{j\in(1....N)}\, fit_j(t) \tag{8}$$

$$worst(t) = max_{j\in(1....N)}\, fit_j(t) \tag{9}$$

In this relation, $M_i(\mathrm{t})$ and $fit_i(t)$ represent the mass and the fitness value of the agent *i* at *t*. According to the gravity law, the overall forces from a set of heavier masses are used to calculate the agent's acceleration (13) by using following equations:

$$F_{ij}^d(t) = \frac{G(t)\, M_j(t)\, M_i(t)}{R_{ij}(t) + \epsilon}\, (\, x_j^d(t) - x_i^d(t)\,) \tag{10}$$

$R_{ij}(t)$ - Euclidian distance between two agents i and j and ε - a small constant. Gravitational constant $G(t)$ is initialized at the beginning of the search and will be reduced with time to control search accuracy as follows:

$$G(t) = \ \ G_O(t) + (\frac{t}{t_{max}})\beta \tag{11}$$

t - Current iterations, t_{max} is the maximum number of iteration. The parameters maximum number of iterations t_{max} , population size N, initial gravitational constant G_O and constant β control the performance of GSA.

$$F_i^d(t) = \sum_{j\in N, j\neq i} rand_j F_{ij}^d(t) \tag{12}$$

$$a_i^d(t) = \ \frac{F_i^d(t)}{M_i(t)} \tag{13}$$

This hybrid is a stochastic algorithm with a feature to select randomly, the important parameters that have an influence on the search procedure. The advantage of implementing PSOGSA is that it avoids getting trapped in local optima, and also improves upon premature convergence probability. It thereby reaches at better optimal solution in a reasonable time. The functionality of both the algorithms is combined and run parallel. The modified velocity equation becomes as stated in Eq. (14).

$$v_i^d(t+1) = wv_i^d(t) + c_1'\ .r.\, a_i^d(t) + c_2'\ .r.\left(p_g^d(t) - x_i^d(t)\right) \tag{14}$$

$v_i^d(t)$ represents velocity of agent i at iteration t, c_1' and c_2' are the positive numbers illustrating the weights of the acceleration terms that guide each particle towards the individual best and swarm best positions respectively. w is the weighing function, r is a random number between 0 and 1, $a_i^d(t)$ is the acceleration of agent i at iteration t , and $p_{\ g}$ is the best solution so far. $d'^d_i(t)$- includes democratic influence of other particles on i^{th} particle in d^{th} dimension.

Each iteration updates the position of particles as (15)

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \tag{15}$$

in which the time interval is equal to 1.0 and thus the velocity vector can be added to the position vector. It is clear that the information produced by all members of the swarm moving with an acceleration guided by GSA, is utilized by the PSO with the purpose of determining new position of each particle, and thus the phrase modified PSOGSA.

OPTIMUM DESIGN RESULTS

In order to evaluate the performance of PSOGSA technique, an example of RC frame structure was studied and optimum design results were presented.

Example: An example consists of a one bay-five storey RC frame, with given geometry and loads was considered. This example has been considered by Moharrami & Grierson (1993) earlier and is shown in Figure 1. The cost ratios of 'steel to concrete' and 'formwork to concrete' have been considered as 50 and 0.6 respectively. Depth of concrete cover has been taken as 63.5 mm. The earlier study is based on optimality criteria method using Lagrangian functions, and the results as compared with present study are given in Table 1. To make the comparison of optimum sectional dimensions and optimum areas of steel, objective function as considered in the previous study has been used here as well. Also, the results of current study are presented in FPS system for comparison with previous study. The permissible compressive stress in concrete and yield stress in steel have been taken as 30 N/mm^2 and 415 N/mm^2 respectively. The depth to width ratio constraint has not been imposed for the present comparison, wherein the minimum dimension of beam and column members has been considered as 12 inches.

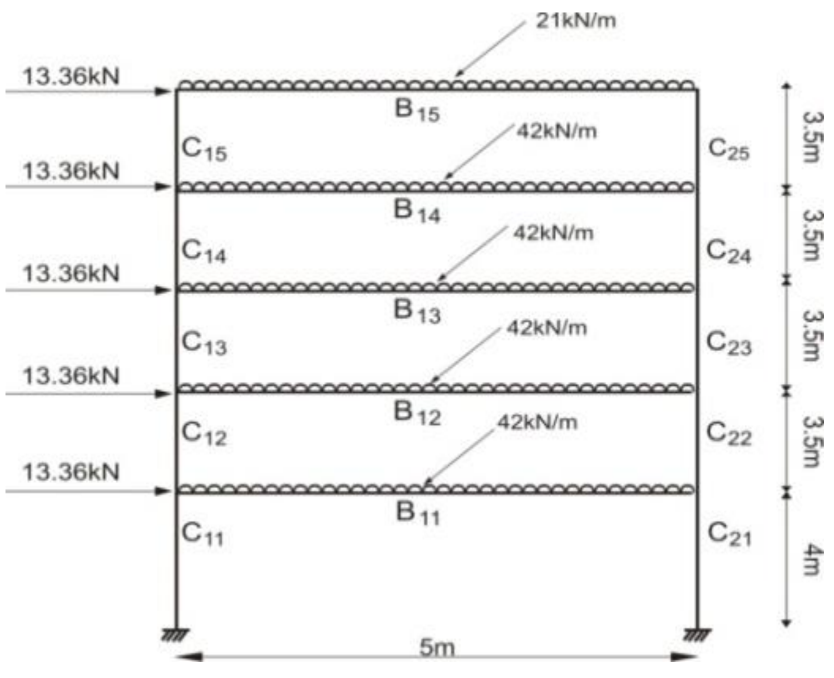

Figure 1 Geometry and loading of one bay-five storey frame

The beam and column design problems have been designed by conventional limit state method as per IS456:2000 and then a set of solution was obtained by applying hybrid particle swarm optimization technique and gravitational search algorithm (PSOGSA). The constant parameters of the algorithm those would be found fine tuned with them are as follows:

$G_O = 100$; $C_1 = 0.5$; $C_2 = 2$; $\beta = 20$ (for PSOGSA)

The population size and maximum number of iterations were also initial input parameters for any population based algorithm and taken as 20 and 500 respectively in this case. The maximum number of iterations was stopping criteria in search of optimum results. It was necessary to define the upper and lower bounds of design variables for the random selection of population.

Table1 Optimal design results of one bay five storey frame

$$Z = (\alpha - 1)V_{st} + V_G + \alpha_1 A_f$$

f_{ck} = 30 N/mm^2, f_y = 415 N/mm^2, $\alpha = 50$, $\alpha_1 = 0.6$, 1 inch = 25 mm

Ex.	MPSOGSA (Present optimum design results)					Past study (Moharrami & Grierson, 1993)[2]				
Member	Depth (in.)	Width (in.)	A_{st} (end) (sq in.)	A_{st} (mid) (sq in.)	Ast (col) (sq in.)	Depth (in.)	Width (in.)	Ast (end) (sq in.)	Ast (mid) (sq in.)	Ast (col) (sq in.)
B_{15}	13.2	12	1.449	0.675		12.0	12	1.901	1.360	-
B_{14}	15.6	12	2.579	1.092		23.41	12	2.782	1.554	-
B_{13}	17.2	12	2.749	1.043		23.41	12	2.782	1.554	-
B_{12}	18.0	12	3.035	1.065		23.41	12	2.782	1.554	-
B_{11}	18.4	12	3.122	1.159		23.41	12	2.782	1.554	-
C_{15}/C_{25}	12.8	12	-	-	1.228	17.95	12	-	-	2.154
C_{14}/C_{24}	12.8	12	-	-	1.228	17.95	12	-	-	2.154
C_{13}/C_{23}	17.2	12	-	-	1.651	17.95	12	-	-	2.154
C_{12}/C_{22}	18.0	12	-	-	1.728	17.95	12		-	2.16
C_{11}/C_{21}	18.8	12	-	-	1.805	17.95	12	-	-	2.16

[a]*12 inches = 300mm*

CONCLUDING REMARKS

In the present study, the analysis of RC frame structure has been performed using direct stiffness approach and the design procedure follows Indian standard IS 456-2000 regulations. Optimum design results are obtained with the use of hybrid technique (PSOGSA). The proposed algorithm overcomes the limitations of two individual algorithms (PSO & GSA) by considering their hybrid, and thereby improves the overall performance. Necessary changes have been incorporated to make the study compatible with earlier study, and to help compare the results. A comparison with other algorithm reveals that reduction in steel area plays a greater role in optimization as compared to reduction in cross sectional area of frame elements particularly verified in design example - by the use of PSOGSA technique. A parameter called 'cost ratio' has been considered for prevalent prices of steel and concrete at a given place so as to impart practical relevance to the study instead of taking it only a piece of pure academic work. Also reduction in steel area and cross sectional area of elements has been achieved in the design of RC frame using this technique.

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REFERENCES

1. ADAMU A, KARIHALOO B L, AND ROZVANY G I N, Minimum cost design of reinforced concrete beams using continuum-type optimality criteria, Structural Optimization, 7(1/2), pp 91-102, 1994.

2. MOHARRAMI H, AND GRIERSON D E, Computer Automated Design of Reinforced Concrete Frameworks, Journal of Structural Engineering, *ASCE*, 119(7), pp 2036-2058, 1993.

3. SAINI B, SEHGAL V K, AND GAMBHIR M L, Genetically optimized artificial neural networks based optimum design of singly and doubly reinforced concrete beams, Asian Journal of Civil Engineering (Building and Housing), 7(6), pp 603-619, 2006.

4. ALQEDRA M, ARAFA M, AND ISMAIL M, Optimum Cost of Prestressed and Reinforced Concrete Beams using Genetic Algorithms. Journal of Artificial Intelligence, 4(1), pp 76-88, 2011.

5. GOVINDARAJ V AND RAMASAMY J V, Optimum design of reinforced continuous beams by genetic algorithms. Computers and Structures, 84, pp 34-48, 2005.

6. COELLO C A, CHRISTIANSEN A D, AND SANTOS F, A simple genetic algorithm for the design of reinforced concrete beams. Engineering with Computers, 13(4), pp 185-196, 1997.

7. LEE C AND AHN J, Flexural design of reinforced concrete frames by genetic algorithm. Journal of Structural Engineering, ASCE, 129(6), pp 762-774, 2003.

8. CAMP C V, PEZESHK S AND HANSSON H, Flexural design of Reinforced Concrete Frames Using a Genetic algorithms, Journal of Structural Engineering ASCE, 129(1), pp 105-115, 2003.

9. RAJEEV S, AND KRISNAMOORTHY C S, Genetic algorithm-based methodology for design optimization of reinforced concrete frames, Computer-Aided Civil Infrastructure Engineering, 13, pp 63–74, 1998.

10. NIMTAWAT A AND NANAKORN P, Simple Particle Swarm Optimization for Solving Beam-Slab Layout Design Problem, Procedia Engineering, 14, pp 1392-1398, 2011.

11. POITRAS G, LEFRANÇOIS G, AND CORMIER G, Optimization of steel floor systems using particle swarm optimization, Journal of Constructional Steel Research, 67, pp1225-1231, 2011.

12. KENNEDY J AND EBERHART R C, Particle swarm optimization, in Proceedings of IEEE international conference on neural networks, 4, pp 1942–1948, 1995.

13. TRELEA I C, The particle swarm optimization algorithm: Convergence analysis and parameter selection, Information Processing Letters 85, pp 317–325, 2003.

14. VALLE Y D, VENAYAGAMOORTHY G K, MOHAGHEGHI S, HERNANDEZ J C AND HARLEY R G, Particle Swarm Optimization: Basic Concepts, Variants and Applications in Power Systems, IEEE Transactions on Evolutionary computation, 12(2), 2008.

15. SHELOKAR P S, SIARRY P, JAYARAMAN V K, AND KULKARNI B D, Particle swarm and ant colony algorithms hybridized for improved continuous optimization, Applied Mathematic Computation, 188, pp 129-42, 2007.

16. KAVEH A AND TALATAHARI S, A Hybrid Particle Swarm and Ant Colony Optimization for Design of Truss Structures, Asian Journal of Civil Engineering (Building and Housing), 9(4), pp 329-348, 2008.

17. MIRJALILI S, AND HASHIM S M, A new hybrid PSOGSA algorithm for function optimization, in International conference on computer and information application (ICCIA), pp 374-377, 2010.
18. RASHEDI E, NEZAMABADI-POUR, H, AND SARYAZDI S, GSA: A gravitational search algorithm, Information Sciences, 179, pp 2232-2248, 2009.

19. IS: 456:2000, Plain and Reinforced Concrete Code of Practice, Bureau of Indian Standards, Manak Bhavan, New Delhi.