CONCRETE MIX DESIGN USING PARTICLE PACKING MODEL

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ABSTRACT. Grading of aggregates has the most significant effect on Packing Density (PD) of aggregates. An important property of multi particle systems is the PD. The packing density is the ratio of the solid volume by the total volume of the container which depends on the placing process. Packing density is new kind of mix design method used to design different types of concrete. To optimize the particle packing density of concrete, the particles should be selected to fill up the voids between large particles with smaller particles, in order to obtain a dense and stiff particle structure. The results obtained by packing density method are compared with Compression Packing (CP) model, Solid Suspension (SS) model and Indian Standards (IS). The optimum bulk density was obtained at proportion of 42%, 18% and 40% for coarse aggregates of maximum size 20mm, 12.5mm and fine aggregates respectively. The compressive strength noticed by PD method and IS are roughly identical, but on the other hand the results obtained with CP model and SS model are somewhere lower than PD.

Keywords: Bulk density, Voids ratio, Packing density, Mixed design.

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INTRODUCTION

There are various methods of proportioning for various types of concrete. Packing density method of mix design is the only mix design method used for proportioning normal concrete, high strength concrete, and no-fines concrete and self-compacting concrete. No adequate literature is available on this method.

The subject of optimizing the concrete composition by selecting the right amounts of various particles has already aroused interest for more than a century. To optimize the particle packing density of concrete, the particles should be selected to fill up the voids between large particles with smaller particles and so on, in order to obtain a dense and stiff particle structure. Most of the early researchers, working on the packing of aggregates, proposed methods to design an ideal particle size distribution.

Geometrically based particle packing models can help to predict the water demand of concrete, and thus the material properties. The cement paste has to fill up the voids between aggregate particles and the "excess" paste will then disperse the aggregate particles to produce a thin coating of paste surrounding each aggregate for lubricating the concrete mix. In general, the higher the packing density of the aggregate, the smaller will be the volume of voids to be filled and larger will be the amount of paste in excess of void for lubrication.

In IS code method of mix design we have curves to decide the water cement ratio whereas in packing density method we don't have such type of co-relation curves available. Here an attempt has made to develop co-relation curves between compressive strength of concrete versus water cement ratio and paste content versus Compressive strength.

Because of superplasticizers and silica fume, it has been possible to produce in laboratory concrete with a cylinder compressive strength of about 150 MPa. On site, the maximum achieved value seems to be 115-120 MPa at 28 days. Such high performance material can be of interest not only for the mechanical strength, but also for some other aspects, like higher modulus, lower creep and shrinkage, or better durability .Much higher strengths have been obtained in the laboratory by using special techniques such as autoclaving, compaction under high pressure, or impregnation with polymers. However, this kind of techniques requires expensive facilities, and is sometimes difficult to apply to full-size elements like beams or slabs. For instance, efficient autoclaving entails penetration of water vapour in the concrete porosity, a difficult goal to match when the concrete piece thickness is higher than a few centimetres. On the other hand, materials incorporating special polymers (like Macro-Defect Free cements, MDF) may display some drawbacks like a high sensitivity to water. Another way of increasing compressive strength is the use of special aggregates like calcined bauxite. Bathe reported on a high-grade DSP (Densified Small Particle) mortar having a compressive strength of 268 MPa. But these aggregates are expensive, so that their industrial interest is limited. Therefore, the research significance of the present project is to see which from such concrete matrix strength level can be obtained by using normal untreated aggregates, cement, silica fume and superplasticizer, when a simple thermal curing is available (comprising only a temperature rise, but neither additional pressure nor humidity). This kind of curing is expected to be feasible as well on site as in ordinary precasting plant. Optimisation is carded out with the help of a mathematical model, together with preliminary testing, in order to reduce the number of tests and to propose a general mix-design methodology. The problem is to find the proportion leading to the best packing density of particles. In solid suspension model, direct equation for finding the packing density can be used by calculating the virtual packing density from the compression packing model and determining the actual packing density of the grain mixture.

In view of the above, it is proposed to develop the concrete mix in the present project work by using three size classes of aggregates viz., 20mm down size, 12.5mm downsize and fine aggregate (4.75mm down size) by changing the proportions to get the maximum density and thereby to calculate the voids content in the concrete mix by assuming 15%, 20% and 25% paste content. It is proposed to design the concrete mixes using the four methods viz., compression packing model, solid suspension model, packing density and I.S. Code as stated above. Finally, it is intended to determine experimentally the 7 days and 28 days cube compressive strength for the concrete mixes for M30 grade using the above four methods and the split tensile strength for the concrete mixes using the packing density and the I.S. code methods of mix design.

Packing Density

An important property of multi particle systems is the packing density. This is defined as the volume fraction of the system occupied by solids. For a given population of grains, it is well known that the packing density, which is the ratio of the solid volume by the total volume of the container, depends on the placing process.

Determination of packing Density

The packing density of individual aggregate in a volume fraction of total aggregate or over all aggregate is determined from its maximum bulk density of mixture and specific gravity from the following relation.

$$packingdensity = \frac{bulkdensity \times weightfraction}{specificgravity}$$

Therefore the equation itself represents that the packing density of the mix of the aggregates is the sum of packing density of individual classes of aggregates. The value of specific gravity should be taken as the average, if the values are differing in the third decimal and if the values are differing in the second decimal individual values should be taken for calculating the packing density and the voids content of the required mix of different classes of aggregates.

Virtual Packing Density

For mono sized grain s we get maximum density by placing one by one in the system we call that as virtual packing density. The virtual packing density is, by convention, the maximum value, which is attainable by placing the grains one by one, without altering their shape. For getting virtual packing density we need to do compact the aggregates as closely as possible to get minimum voids in that size class. For round aggregates (i.e., the spherical) monodisperse arrangement of spheres may achieve a packing density of 0.74 (compacted arrangement).It can be achieved experimentally from the heavy compaction of a aggregate class by determining its bulk density, specific gravity.

Actual Packing Density

Actual packing density is differ from theoretical packing density, since the actual packing density depends upon many factors like size of aggregates, method of compaction, nature of aggregates. Actual packing density is always less than the theoretical packing density and it depends upon the amount of compaction, the shape of the aggregate chosen.

Models of the Packing Density of Grain Mixtures

Linear Packing Density Model for Grain Mixtures (LPDM)

In 1951, Mooney developed a model for predicting the viscosity of multimodal suspensions of non-reactive particles [15]. We have shown that this model can be used as a packing model, just by searching the liquid proportion leading to infinite viscosity [16]. A large number of dry packing experiments have allowed a calibration of this packing model, either for crushed or rounded particles [17]. Equations of the Linear Packing Density Model (LPDM) are the following:

c = min(c (t)) for y (t)) 0 with

$$c(t) = \frac{\alpha(t)}{1 - \int y(x) f\left(\frac{x}{t}\right) dx - [1 - \alpha(t)] \int y(x) g(t/x) dx}$$

$$(z) = 0.7(1 - z) + 0.3(1 - z)^{12}$$

$$g(z) = (1 - z)^{1.3}$$

where c is the packing density, t the size of the grains, y (t) the voluminal size distribution of the grain mixture (having a unit integral $\int_d^D y(x)dx = 1$); d and D are respectively the minimum and d maximum sizes of grains, α (t) is the specific packing density of the t-class, f (z) is the loosening effect function and g(z) is the wall effect function. These functions, describing the binary interactions between size classes, are expected to be universal, while y (t) and α (t) depend on the considered granular mix, and can be measured. LPDM has shown good performances in predicting optimal proportions of superplasticized cementitious materials (cement pastes [18], mortars and concretes [17]). But it suffered from an original defect, owing to its linear nature: curves giving relationship between packing density and proportions exhibit angular points in the vicinity of optimal values. Such a feature does not appear in practice. This is why a better model is needed.

Compression Packing Model (Extension of L.P.D.M)

In 1986, Stovall proposed an equation for determining the actual packing density of the mixture by introducing a factor called compression index, (k).

$$\gamma = \frac{\beta}{1 - \sum_{j=1}^{I=1} 1 - \beta I + Bij\left(1 - \frac{1}{\beta j}\right) yi - \sum_{j=i+1}^{n} [1 - \frac{\alpha ij\ \beta i}{\beta j}]}$$

The above equation gives packing density of i size class of n size classes which was dominant in the mixture. In the above equation γ_I represents the packing density of I class aggregates, β_i represents the virtual packing density of I class which we can get by compacting them alone , y_j

represents the volumetric fraction of j class in the mixture and aij and bij represents the interaction coefficients describing the loosening effect and wall effect respectively.
The interaction coefficients that are described in the above can be determined from
the from equations 3.2.3.1 and 3.2.3.2

$$aij = \sqrt{1 - (1 - \frac{di}{dj})^{1.02}}$$

$$bij = 1- [1-\frac{di}{dj}]^{1.5}$$

Where di and dj are diameters of the granular classes i and j as defined by sieve sizes in which aij and bji respectively designate the coefficient of the loosening effect exerted by the grains of rank j on those of rank i (j > i) and the wall effect of the grains of classi on the grains of rank j (j < i), with d1 > di > dn

Where aij=1 when dj > di and
bij=1, when di > dj

The real packing density is lower than the virtual packing density and it depends on the applied compaction energy. To determine the real packing density a scalar (i.e., the compaction index K) is introduced, which depends on compaction only. As k tends to infinity, the theoretical packing density (αt) tends to virtual packing density (β). The packing density αt can be determined indirectly from equation (L.P.D.M).

$$K=\sum_{i=1}^{n} K_i=\sum_{i=1}^{n} \frac{Yt/\beta t}{\frac{1}{\alpha t}-1/\gamma t}$$

Table 1 Indicative Packing Index for Setting Modes of Dry Mixtures

Process	k (compaction Index)
Pouring	4.1
Sticking With A Rod	4.5
Vibration	4.75
Vibration + Compression 10 Kpa	9

The compaction index k for dry rodding procedure is 4.5 (De Lerrard 1999).

A New Packing Model (Solid Suspension Model)

In this last development, we have come back to Mooney's original model, by considering a random packing of particles like a suspension of high but finite viscosity. Therefore, the reference specific packing densities are shifted towards higher values.
For example, it is well known that a mono disperse arrangement of spheres may achieve a packing density of 0.74 (compact hexagonal arrangement), while a random packing of the same particles gives no more than 0.64[19]. Following model solid suspension model has been made in relationship with the mono disperse suspension ϕ and its relative viscosity n_r

$$n_r=\exp\left(\frac{2.5}{(\frac{1}{\phi}-\frac{1}{\beta})}\right)$$

Here we will assume that represents the maximum packing density, while ϕ is the random one with β=0.74 and ϕ =0.64, one have n_r=1.36*105 = nr^{ref}.

Then with the same formalism as in LPDM, the packing density for any grain mixture is given in the following implicit equation:

$$Nr^{ref}=\exp[\int_d^D \frac{2.5y(t)}{(\frac{1}{c}-\frac{1}{c(t)})}\,dt$$

$$C(t)=\frac{\beta i}{1-\sum_{j=1}^{i=1} 1-\beta i+bij\,\beta i\left(1-\frac{1}{\beta i}\right)yi-\sum_{j=i+1}^{n}\left[1-\frac{aij\,\beta i}{\beta j}\right]yj}$$

Where β (t) is the virtual packing density of p size grains calculated from the experimental (random arrangement) one with the next equation.

$$N_r^{ref}=\exp[\frac{2.5}{\frac{1}{\alpha(t)}-\frac{1}{\beta(t)}}]_{\text{for} d \le t \le D}$$

When a t-size class consists of N different types of grains, each one characterized for i=1 to N, by its own partial volume yi(t) (with $\sum_{i=1}^{N} y_i(t) = 1$) and $\beta_i(t)=\frac{1}{\beta(t)}=\sum_{i=1}^{N}\frac{yi(t)}{\beta i(t)}$.In fact, as they have the same size, the different types of grains are supposed to have no influence on the packing of the other ones. According to that, the solid volume $y_i(t)$ occupies the volume. Then, the solid volume $\sum_{i=1}^{N} y_i(t)=1$) is contained in the total volume $\sum_{i=1}^{N}\frac{yi(t)}{\beta i(t)}$which justifies the expression β (t).

Packing Density Method (A Practical Method of Finding the Packing Density) [22]

In this method he calculated practically by determining the bulk density of various proportions of coarse aggregate and the fine aggregate.

The packing density of aggregate mixture is defined as the solid volume in a unit total volume. The aim of obtaining packing density is to combine aggregate particles in order to minimize the porosity, which allows the use of least possible amount of binder.

Two size fractions of coarse aggregates were selected for the study i.e., 20mm and12.5mm down size. The values of bulk density of the coarse aggregates (20mm and12.5mm size) were first determined separately. The coarse aggregate 20mm and12.5mm were mixed in different proportions by mass, such as 90:10, 80:20, 70:30 and60:40 etc., and the bulk density of each mixture is determined. Addition of smaller size aggregate (12.5mm down size) increases the bulk density. However a stage is reached when the bulk density of coarse aggregate mixture, which instead of increasing, decreases again.

Total packing density of the mixture is sum of packing density of 20mm,12.5mm and fine aggregate i.e., equal to the ratio of bulk density of mixture to specific gravity of individual aggregate (20mm : 12.5mm : fine aggregate). The value of specific gravity should be taken as average, if the values are differing in third decimal and if the values are differing in second decimal, the individual values should be taken for calculating packing density and voids content. The optimum bulk density was obtained at proportion of 42% coarse aggregates (20mm downsize), 18% coarse aggregates (12.5mm downsize) and 40% fine aggregates. Large number of trial casting were carried out for each grade of concrete(i.e., M20, M25, M30, M35 and M40) with different water cement ratio and three paste contents in excess of void content. To finalize mix proportions using packing density method flow table tests were carried out to decide water cement ratio and paste content in excess of void content for each grade of concrete. The finalized mix proportion for each grade of concrete was used to cast the cube specimens for 7 days and 28 days curing age. The cube compressive strength a result obtained by packing density and IS code method are nearly same. The co-relation curve was plotted for packing density results alone and also combining the results of packing density and IS code methods. The co-relation curves were plotted between compressive strength vs water cement ratio at 7 and 28 days curing age and compressive strength vs paste content at 7 and 28days curing age. Very good co-relation is obtained with a co-relation co-efficient of0.953 (minimum) to 0.998 (maximum). These curves can be used to decide the water cement ratio and paste content for the specified grade of concrete in case of packing density method thus reducing the material and time involved in trial testing.

DISCUSSIONS

We have taken three classes of aggregates as C.A1 (20mm down and 12.5mm retained) C.A2 (12.5mm down and 4.75mm retained) and fine aggregate 4.75 mm down and determined the bulk density by varying proportions as first C.A1 and C.A2 and in second C.A1, C.A2&F.A.

Determination of Virtual Packing Density

For mono sized grains we get maximum density by placing one by one in the system we call that as virtual packing density. The virtual packing density is, by convention, the maximum value, which is attainable by placing the grains one by one, without altering their shape. For getting virtual packing density we need to do compact the aggregates as closely as possible to get minimum voids in that size class. For round aggregates (i.e., the spherical) mono disperse arrangement of spheres may achieve a packing density of 0.74 (compacted arrangement). It can be achieved experimentally from the heavy compaction of a aggregate class by determining its bulk density, specific gravity. For achieving virtual packing density we need to find the bulk density of the monosized disperse particles by doing heavy compaction and the results are shown below.

Table 2 Bulk Density of Full Compacted Coarse Aggregat*e*

Sl.no.	W_1(kg)	W_2(kg)	(kg)($w_3=w_2-w_1$)	(w_4)kg	$\gamma= w_3/(w_4-w_1)$	γ Average(kg/m^3)
C.A1	11.91	36.5	24.59	26.91	1639.333	
	11.91	36.9	25.08	26.91	1672	1651.933
	11.91	36.577	24.667	26.91	1644.467	
C.A2	3.55	8.2	4.65	6.55	1550	
	3.55	8.23	4.68	6.55	1560	1564.444
	3.55	8.3	4.75	6.55	1583.333	

And for fine aggregate, the bulk density fully compacted is 1651.778 kg/m3

Table 3 Bulk Density of the 20 mm down and 12.5mm down Size with Varying Proportions in the Mix

S.No	w1(kg)	w2(kg)	(w3=w2-w1)	(w4)	γ= w3/(w4w1)	Proportion of 12.5mm pass
1	11.91	35.85	23.94	26.91	1.596	0.1
2	11.91	36.255	24.345	26.91	1.623	0.2

3	11.91	36.99	25.08	26.91	1.672	0.3
4	11.91	36.63	24.72	26.91	1.648	0.4
5	11.91	34.56	22.65	26.91	1.51	0.5
6	11.91	34.11	22.2	26.91	1.48	0.6

Initial weight of cylinder = W_1(kg)
Weight of cylinder with aggregate = W_2(kg)
Net wt. of aggregate = (w_3=w_2-w_1)kg
Wt. of cylinder with water = (w_4)kg
Bulk density(kg/m_3)γ= w_3/(w_4-w_1)

Calculation of Maximum Bulk Density by Changing Proportions of C.A1;C.A2 [11]

Two size fractions of coarse aggregates were selected for the study i.e., 20mm and12.5mm down size. The values of bulk density of the coarse aggregates (20mm and12.5mm size) were first determined separately. The coarse aggregate 20mm and12.5mm were mixed in different proportions by mass, such as 90:10, 80:20, 70:30 and60:40 etc., and the bulk density of each mixture is determined. Addition of smaller size aggregate (12.5mm down size) increases the bulk density. However a stage is reached when the bulk density of coarse aggregate mixture, which instead of increasing, decreases again. The results of Bulk density of coarse aggregate fractions (20mm and 12.5mm) are plotted.

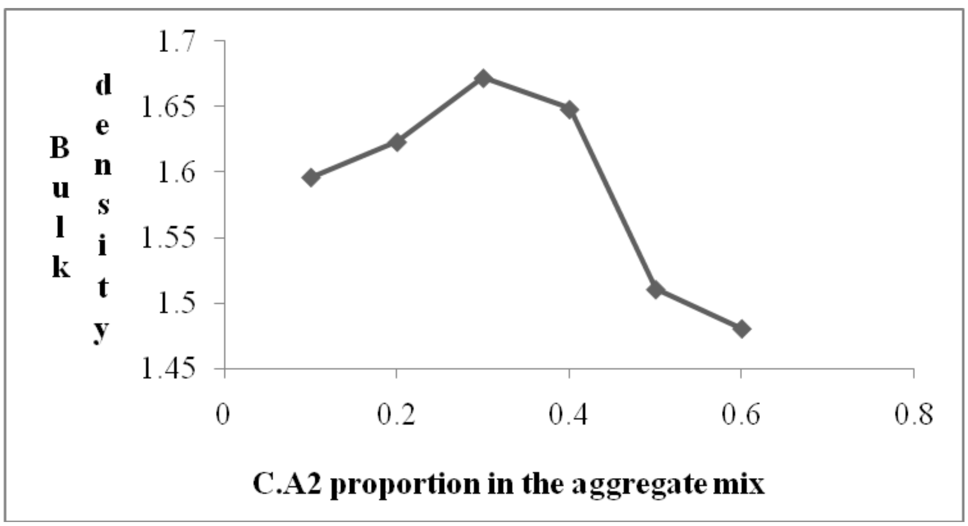

Figure 1 Bulk density vs C.A2 proportion in the aggregate mix

Calculation of Maximum Bulk Density by Proportionating the Fine Aggregate [12]

Increase in fine aggregate particles leads to decrease in void content thus increases the bulk density. The replacement of fine aggregates in the total coarse aggregates (20mmand 12.5mm down size in the proportion 70:30) in the ratio of 80 : 20, 70 : 30, 60 : 40,50 : 50.By increasing the finer content the bulk density increases up to a maximum extent after which it again reduces. Thus the proportion obtained for maximum bulk density is fixed as total coarse aggregates: fine

aggregates i.e., 60: 40. Total coarse aggregate proportion i.e., 20 mm: 12.5 mm is fixed as 70: 30 as mentioned earlier.
Therefore proportions of these aggregates i.e., coarse aggregates 20 mm: coarse aggregates 12.5 mm: fine aggregates is 42: 18: 40. The bulk density, packing density and voids ratio are plotted against the mass fraction of coarse aggregate are plotted.

Table 4: Bulk Density of Aggregates Mix and the Proportion of Fine Aggregate in the Mix

Proportion of fine aggregate	Bulk density(kg/m3)
0.2	1.8
0.3	1.83
0.4	1.96
0.5	1.71

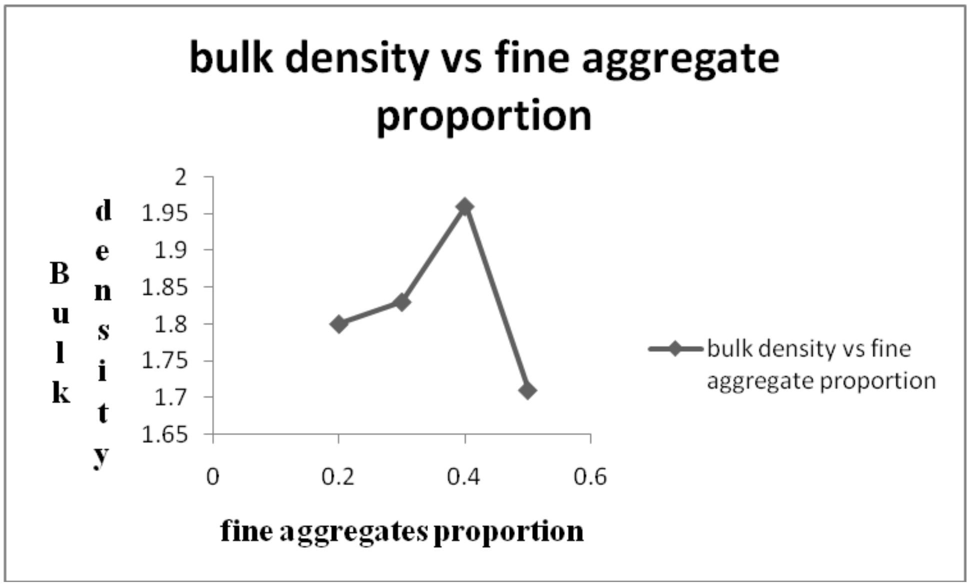

Figure 2 Bulk density vs fine aggregates

Specific Gravity and Water Absorption

Calculations
Weight of saturated aggregate suspended in the water with basket = w1kg
Weight of basket suspended in water = w2kg
Weight of saturated aggregate in water = ws = w1-w2kg
Weight of saturated surface dry aggregate in air = w4kg
Weight of water equal to volume of aggregate = (w3-ws) kg

Specific gravity = $\frac{dry\ weight\ of\ aggregate}{WEIGHT\ OF\ EQUAL\ VOLUME\ OF\ WATER}$

$= \frac{W4}{W3 - WS}$

Apparent specific gravity = $\frac{dry\ weight\ of\ aggregate}{weight\ of\ equal\ volume\ excluding\ air\ voids\ in\ aggregate}$

$$=\frac{w4}{w4-ws}$$

Water absorption= percentage by weight of water observed in terms of Owen dry weight of aggregate $= \frac{w3-w4}{w4} \times 100$

Table 5 Specific Gravity and Water Absorption of Coarse Aggregate

w1	w2	w3	w4	Specific Gravity	Apparent S.G	Water Absorption	
C.A1	2.25	0.88	2.18	2.1722	2.6913580 25	2.7078035 4	0.2898165 14
C.A2	2.06	0.846	1.974	1.9663	2.5973684 21	2.6137179 32	0.2964539 01

For fine aggregate specific gravity is 2.68 and water absorption is 2%

Sieve Analysis of Fine and Coarse Aggregates [11]

We have to arrange the sieves in the descending order of the sieves and we have to sieve them foe around 5 minutes and after sieving we have to take percentage weight retained on each sieve and graph has to be plotted between the percentage finer (by mass) against the sieve size. And as per the I.S code we have to check how much amount is passing through each sieve. And based on the percentage passing we have to decide the zone of the sand and fineness modulus which are very essential i9n the I.S method of mix design I.S 10262.

Table 6 Results of Fine Aggregate Sieve Analysis

S.No	IS sieve size	Retained on IS sieve	Retained (%)	PERCENT FINER
1	4.75	0	0	100
2	2.36	0.097	9.7	90.3
3	2	0.067	6.7	93.3
4	1	0.26	26	74
5	0.425	0.33	33	67
6	0.3	0.127	12.7	87.3
7	0.15	0.113	11.3	88.7
8	0.075	0.006	0.6	99.4
9	0	1	100	0

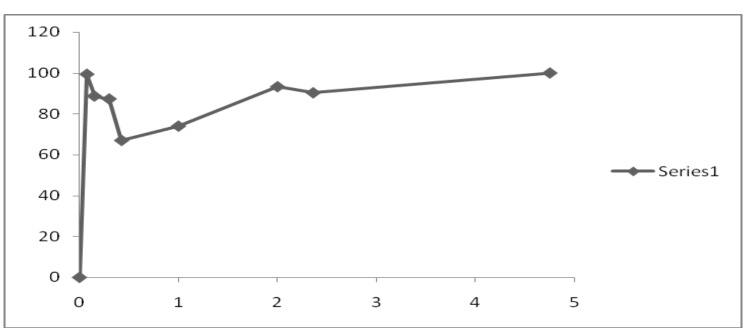

Figure 3 Sieve Analysis of Fine Aggregate

Water Content Determination

Slump cone test (workability test)
By workability test we have tested for M30grade of concrete for w/c ratio 0.4; 0.35; 0.3and we have got for a w/c of 0.4 we got good workable concrete.

Packing density method [22]

The optimum bulk density was obtained at proportion of 42% coarse aggregates (20mm downsize), 18% coarse aggregates (12.5mm downsize) and 40% fine aggregates

$$\text{P.D (20-12.5)} = \frac{1.9\times0.42}{2.69}$$

$$\text{P.D (12.5-4.75)} = \frac{1.9\times0.18}{2.59}$$

$$\text{P.D (4.75 down)} = \frac{1.9\times0.4}{2.68}$$

Total packing density of the mixture = P.D (20-12.5) +P.D (12.5-4.75)+P.D(4.75 down)
= 0.7342

Compression packing model

Virtual packing density

$$\beta_i\text{ (12.5-20)} = \frac{1.6519\times0.42}{2.691} = 0.258$$

$$\beta_i\text{(4.75-12.5)} = \frac{1.564444\times0.18}{2.597} = 0.108$$

$$\beta_i\text{(4.75 down size)} = \frac{1.6517\times0.4}{2.68} = 0.247$$

Total packing density $\gamma i = \frac{\beta i}{1-\sum_{i=1}^{i-1} 1-\beta+bij\beta i\left(1-\frac{1}{\beta i}\right)\gamma i-\sum_{j=i+1}^{n}\left(1-\frac{aij\beta i}{\beta j}\right)\gamma j}$

Packing density of (12.5-20) = $\frac{}{1-\left(1-0.258+1\times0.258\left(1-\frac{1}{0.108}\right)\right)\times0.18-[1-0.258+1\times0.258\times\left(1-\frac{1}{0.247}\right)]\times0.4} = 0.204$

Packing density of (4.75-12.5) = $\frac{0.108}{1-\left[1-0.108+0.512\times0.108\times\left(1-\frac{1}{0.247}\right)\right]\times0.4-[1-\frac{1\times0.108}{0.258}]\times0.18} = 0.178$

Packing density (4.75 down size)= $\frac{0.247}{1-\left(1-\frac{1\times0.247}{0.258}\right)\times0.42-(1-\frac{1\times0.247}{0.108})\times0.18} = 0.203$

Therefore total virtual packing density =0.203+0.203+0.178 = 0.584
Actual packing density:

Packing density of 12.5mm-20mm $\quad 4.5 = \frac{\frac{0.42}{0.258}}{\frac{1}{x}-\frac{1}{0.204}}$

X= 0193

Packing density 12.5mm-4.75mm $\quad 4.5 = \frac{\frac{0.018}{0.108}}{\frac{1}{x}-\frac{1}{0.0182}}$

X = 0.167

Packing density 4.75mm down size $\quad 4.5 = \frac{\frac{0.4}{0.47}}{\frac{1}{x}-\frac{1}{0.3147}}$

X = 0.187

Total packing density=0.193+0.167+0.187 = 0.547

Solid Suspension Model

Nr^{ref}= exp[$\frac{2.5}{\frac{1}{\alpha(t)}-\frac{1}{\beta(t)}}$

Packing density (20-12.5) $\frac{2.5}{\frac{1}{x}-\frac{1}{0.19}}$ = 0.211

x=0.185

Similarly for (4.75-12.5) and (4.75 down size) are 0.171 and 0.186 respectively
Therefore total packing density = 0.55

Table 7 Packing Densities and void content of Various Methods

S.No	Method	Packing Density Calculated	Percentage Voids Content
1	Packing Density Method	0.7342	0.2658
2	Compression Packing Model	0.547	0.453
3	Solid Suspension Model	0.55	0.45

Concrete Mix Design (Normal) Using Packing Density Method [22]

Determination of Paste content for M30 Grade Concrete: Minimum paste content is sum of the void content in combined aggregate and excess paste over and above it to coat the aggregate particle. Meaning of minimum paste content can be explained as, a concrete mix containing minimum paste content should be cohesive, free from segregation and bleeding. Flow table test were carried out to decide the minimum paste contents required to form the workable mix for different W/C ratio and different paste content in excess of void content.
Voids content = 1 – 0.7342 = 0.2658
Assuming paste content as 10% in excess of void content, detailed calculations to obtain all the ingredients of concrete such as coarse aggregate 20mm, 12.5mm, fine aggregate, cement and water content is given below.
Paste content 20% in excess of void content
Paste content = 0.2658+ 0.2x 0.2658= 0.319
Volume of aggregates = 1 – 0.319 =0.681
Totalsolid volume of aggregates =

$$\frac{\text{weight fraction of 20mm}}{\text{specific gravity}}+\frac{\text{weight fraction of 12.5mm}}{\text{specific gravity}}+\frac{\text{weight fraction of fine aggregate}}{\text{specific gravity}}$$

$$=\frac{0.42}{2.691}+\frac{0.18}{2.597}+\frac{0.4}{0.268} = 0.37538$$

Weight of (20-12.5) aggregate = $\frac{0.681\times0.42\times1000}{0.3754}$ = 761.91kg/m3

Weight of 12.5mm passing aggregate = $\frac{0.681\times0.18\times1000}{0.3754}$ = 326.5317 kg/m3

Weight of fine aggregate = $\frac{0.681\times0.4\times1000}{0.3754}$ = 725.626 kg/m3

For M30 w/c = 0.4 w = 0.4c

Total paste = c+w = $\frac{c}{2.9}+\frac{0.4c}{1}$= 0.7448c

Cement content = $\frac{0.319\times1000}{0.7448}$ = 428.188 kg/m^3

Water content = 0.4×c =171.28 kg/m^3
Therefore C.A1= 761.91 kg/m^3 C.A2 =326.5317 kg/m^3 F.A= 725.626 kg/m^3
Cement = 428.188kg/m^3 Water Content = 171.28 kg/m^3
Similarly we have to do for Compression Packing Model and Solid Suspension Model

I.S Code Method of Mix Design [13]

Strength of Concrete: 30 MPa
Standard deviation: 5MPa
Target mean strength = 38.25MPa
Selection of water cement ratio
Maximum water cement ratio = 0.6
Adopted water cement ratio = 0.4
Selection of water content
For 20mm aggregate maximum water content = 186 liters
Water content for required slump = 197.16 liters
Reduction = 0 % (since zone 2 sand)
Calculation of cement content
Water cement ratio = 0.4
Cement content = 492.9 kg/m3
Correction for aggregate size = 0
Minimum cement content = 240 kg/m3
Adopted cement content = 492.9 kg/m3
Proportion of volume of aggregate
Volume of coarse aggregate = 0.62
Correction
Based on w/c ratio = 0.02
Based on placement by hand = 0
Hence volume
Coarse aggregate = 0.64
Fine aggregate = 0.36
Mix calculations
Volume of concrete = 1m3
Volume of cement = 0.17 m3
Volume of water = 0.2 m3
Volume of chemical admixture = 0
Volume of aggregate (C A+F A) = 0.63 m3
Mass of coarse aggregate (20 mm-12.5mm) = 754.187 kg
Mass of coarse aggregate (12.5mm down) = 323.223 kg
Mass of fine aggregate = 610.6kg
Water correction
Extra quantity of water to be added
Coarse aggregate = 3.02 kg
Fine aggregate = 12.21 kg
Quantity of water to be deducted

Coarse aggregate = 0kg
Fine aggregate = 0kg
Mix proportions for trial
Cement = 493kg/ m3
Water = 212 kg/ m3
Fine aggregate = 598 kg/ m3
Coarse aggregate (20-12.5) = 751.8 kg/ m3
Coarse aggregate (12.5 down) = 322.2 kg/ m3
Chemical admixture = 0
Water cement ratio = 0.43
Mix proportions 1: 1.34: 2.38

Table 8 Various Methods and Mix Contents for 25% Paste Content

S.No.	Method	Packing Density	Water (kg/m3)	Cement(kg/m3)	C.A1(kg/m3)	C.A2(kg/m3)	F.A(kg/m3)
1	Packing	0.732	171.28	428.188	761.91	326.5317	725.626
2	CPM	0.55	257.8	644.47	514.65	220.56	490.14
3	SSM	0.547	295.384	738.46	503.36	215.77	479.488
4	I.S.Code	*	212	493	751.8	322.2	598

Table 9 Various Methods and Mix Contents for 15% Paste Content

S.No.	Method	Packing Density	Water (kg/m3)	Cement(kg/m3)	C.A1(kg/m3)	C.A2(kg/m3)	F.A(kg/m3)
1	Packing	0.732	169.641	394.5147	776.820	332.923	739.829
2	CPM	0.55	284.428	661.461	539.829	231.3932	514.418
3	SSM	0.547	313.009	729.929	487.799	209.052	469.598
4	I.S.Code	*	212	493	751.8	322.2	598

Table 10 Various Methods and Mix Contents for 20% Paste Content

S.No.	Method	Packing Density	Water (kg/m3)	Cement(kg/m3)	C.A1(kg/m3)	C.A2(kg/m3)	F.A(kg/m3)
1	Packing	0.732	184.392	428.82	747.083	320.178	711.507
2	CPM	0.55	324.75	755.236	489.477	209.776	466.169
3	SSM	0.547	339.185	788.802	461.507	197.789	439.531
4	I.S.Code	*	212	493	751.8	322.2	598

The above calculations are carried out by assuming the paste content 15%, 20%, 25% excess in void %, there by calculating the total solid volume remaining in the mix. The solid volume includes the C.A1, CA2 and the fine aggregate. After calculating the aggregates content in the mix, cement and the water content is calculated from the assumed 15%, 20%, and 25% paste content in excess of the void content.

Table 11 Mix Proportions in Packing Density with Variation in Paste Content

Packing density	cement	sand	aggregate
15%	1	1.875	2.813
20%	1	1.694	2.588
25%	1	1.659	2.488

Table 12 Mix Proportions in C.P.M with Variation in Paste Content

Packing density	Cement	Sand	Aggregate
15%	1	0.774	1.116
20%	1	0.76	1.14
25%	1	0.617	0.926

Table 13 Mix Proportions in S.S.M with Variation in Paste Content

Packing density	cement	sand	aggregate
15%	1	0.643	0.9546
20%	1	0.65	0.97
25%	1	0.557	0.8358

Testing of Compressive Strength of Concrete (7 Days & 28 Days) [14]

For cube compressive strength of 7 days and 28 days we have casted 3 cubes for each paste content (i.e., for 15%, 20% and 25%) and for each method totally 36 cubes for 28 days and also for 7 days strength.

Table 14 Cube Compressive Strength 28 Days for All the Four Methods

Method of mix/ Paste content in excess of void content	Three Cubes Average 28 days strength in N/mm^2		
	15%	20%	25%
I.S. Code	33.16	33.16	33.16
packing density method	31.513	35.88	42.7204
CPM	28.56	34.23	36.54
SSM	30.66	35.66	38.35

Table 15 Cube Compressive Strength 7 Days for All the Four Methods

Method of mix/ Paste content in	7 days cube compressive strength in N/mm^2		
	15%	20%	25%

of void content excess			
P.D Practical	20.48345	24.3	28.62267
SSM	20.8488	24.2488	25.8285
CPM 19.4208 23.7264 24.8472			
I.S code	22.2172	22.2172	22.2172

Comparison of Packing Density with Raj et al (IOSR JMCE Volume 11, Issue2 Ver1 Mar-Apr, 2014, PP34-46)

Table 16 Packing Density 28 Days Strength Comparison with IOSR-JMCE 2014

Packing density / paste content in excess of void content	15%	20%	25%
28 days experimental	31.513	35.88	42.7204
Raj et al(IOSR JMCE Volume 11, Issue2 Ver1 Mar-Apr, 2014, PP34-46)	39.065	44.48	49.895

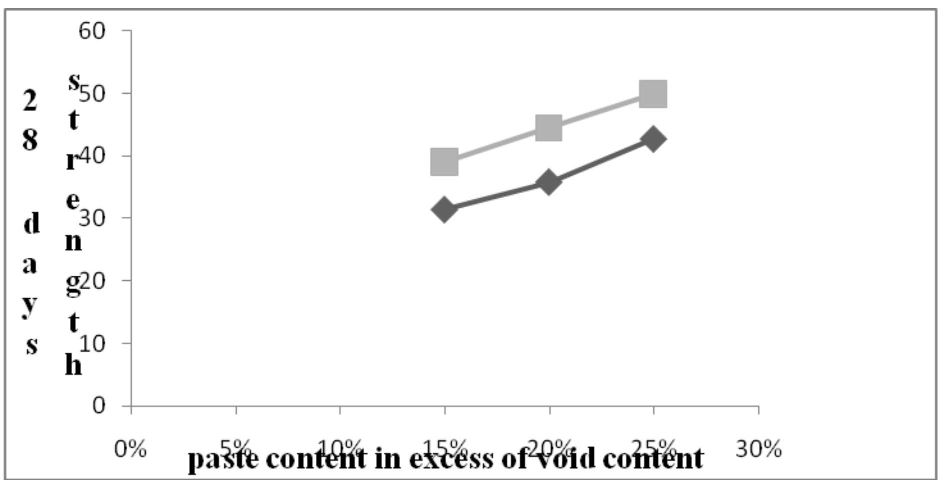

Figure 4 Comparison of 28 Days Cube Compressive Strength between the Packing Density (Experimental) with the Raj et al (IOSR JMCE Volume 11, Issue2 Ver1 Mar-Apr, 2014, PP34-46

The experiments that we have carried out in packing density method is showing a linear relationship between the cube compressive strength and the paste content in excess of void% and the trend line is $y = 112.07x + 14.29$ with the co-relation co-efficient(r) as 0.991and with the standard error 1.009761322.

The above equation is found to be similar with the linear equation $y = 108.3x + 22.82$ which was represented by Raj et al (IOSR JMCE Volume 11, Issue2 Ver1 Mar-Apr, 2014, PP34-46and with the correlation coefficient and standard error as 0.998 and 0.566 respectively.

Table 17 Packing Density 7 Days Strength Comparison with Raj et al (IOSR JMCE Volume 11, Issue2 Ver1 Mar-Apr, 2014, PP34-46

Packing density method/ paste content	7 days strength 15%	20%	25%
7 days practical	20.48345	24.3	28.62267
Raj et al(IOSR JMCE Volume 11, Issue2 Ver1 Mar-Apr, 2014, PP34-46	26.5642	30.2464	33.42965

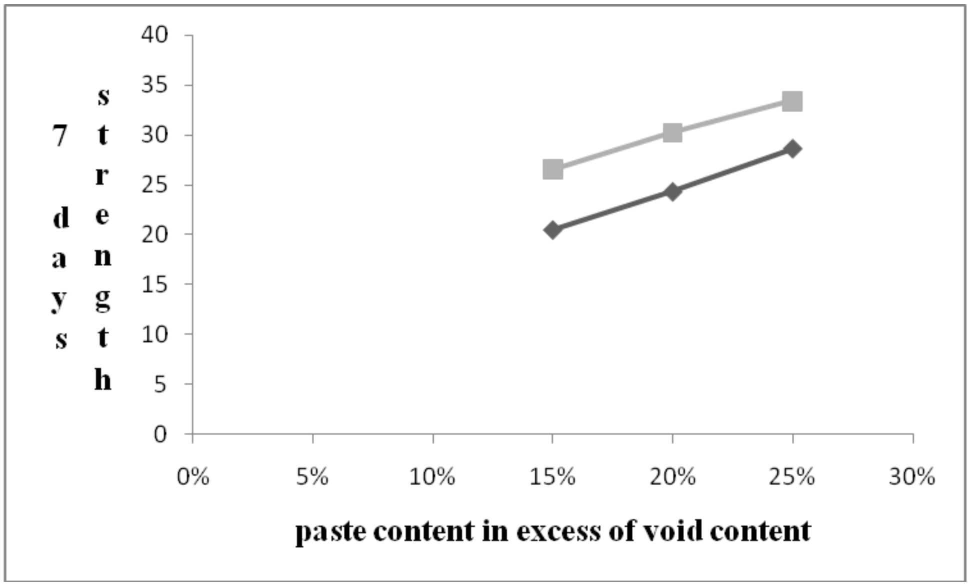

Figure 5 Comparison of 7 Days Cube Compressive Strength between the Packing Density (Experimental) with the Raj et al (IOSR JMCE Volume 11, Issue2 Ver1 Mar-Apr, 2014, PP34-46

The experiments that we have carried out in packing density method is showing a linear relationship between the cube compressive strength and the paste content in excess of void% and the trend line is y = 81.392x + 8.1903 with the correlation coefficient and standard error as 0.999356171 and 0.206622625 respectively,. The above equation is found to be similar linear equation y = 68.654x + 16.349 which was represented by Raj et al(IOSR JMCE Volume 11, Issue2 Ver1 Mar-Apr, 2014, PP34-46 with the correlation coefficient and standard error as 0.995 and 0.823 respectively.

Comparison of Solid Suspension and Compression Packing Models

Table 18 SSM & CPM 28 Days Strength Comparison

Method/paste content	15%	20%	25%
cpm 28 days cube Compressive strength in	28.56	34.23	36.54

N/mm2			
ssm 28 days cube Compressive strength in N/mm2	30.66	35.66	38.35

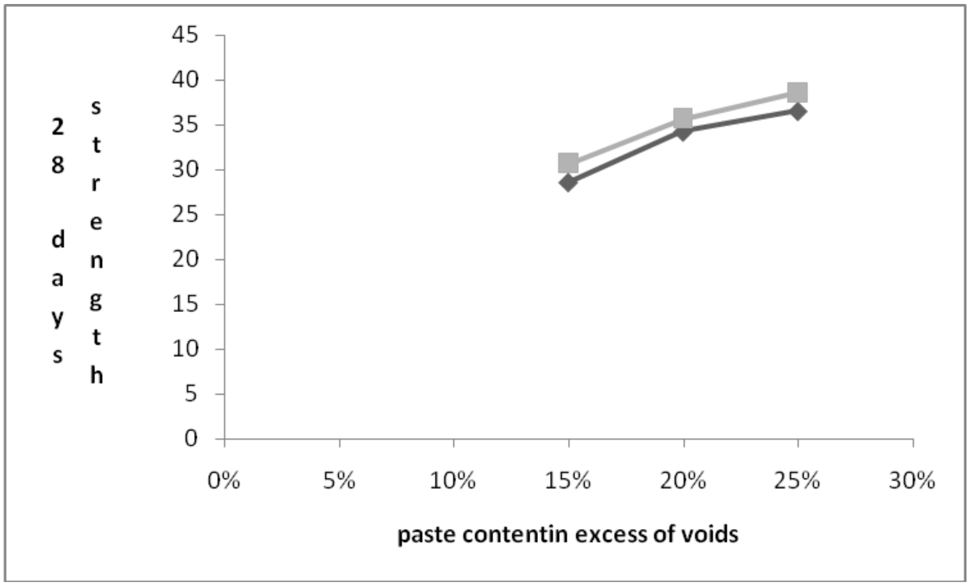

Figure 6 Comparison between SSM & CPM 28 Days Cube Compressive Strength

The SSM (analytical method) which is the extension of the linear packing density model is found to be giving good results than the CPM and it is showing a polynomial trend of order 2 and the equation is y = -462x2 + 261.7x + 1.8 and the CPM equation is is y = -672x2 + 348.6x - 8.61.

Table 19 SSM & CPM 7 Days Strength Comparison

Method/ paste content in excess of void content	15%	20%	25%
s.s.m cube compressive strength of 7 days	20.8488	24.2488	25.8285
c.p.m cube compressive strength of 7 days	19.4208	23.7264	24.8472

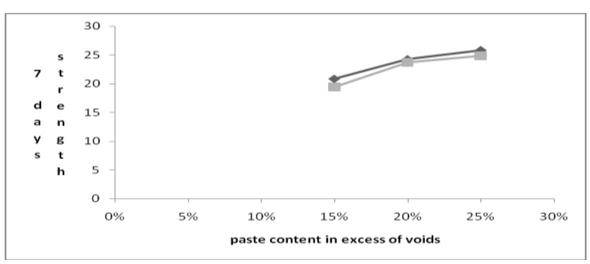

Figure 7 Comparison between SSM & CPM 7 Days Cube Compressive Strength

The SSM (analytical method) which is the extension of the linear packing density model is found to be giving good results than the CPM and it is showing a polynomial trend with order 2 and the equation is y = -364.06x2 + 195.42x - 0.273 and the CPM equation is y = -636.96x2 + 309.05x - 12.605

Testing Of Split Tensile Strength of Concrete

For split tensile strength of 28 days 9 cylinders had been casted for packing density method for 15%, 20%, 25% paste content and we have got approximately 0.689 times the 28 days cube compressive strength of concrete.

Table 20 Split Tensile Strength 28 Days with Varying Paste Content

PACKING DENSITY/PASTE CONTENT	SPLIT TENSILE STRENGTH in N/mm2
15%	21.712457
20%	24.72132
25%	29.4343556

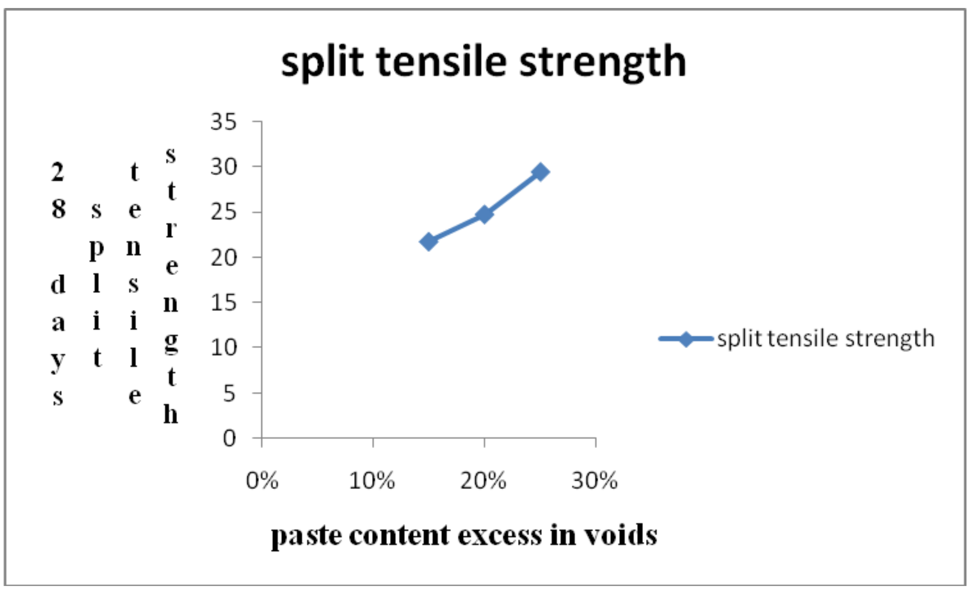

Figure 8 Split Tensile Strength 28 Days with Varying Paste Content

The split tensile strength of the packing density is showing a linear variation with the paste content and the equation of the trend line is y = 77.219x + 9.8456 and the correlation co-efficient is 0.992 and with standard error is 0.695728572.

Comparison of Cube Compressive Strength and Split Tensile Strength of Packing Density Method 28 Days

Table 21 Comparison of Cube Compressive and Split Tensile Strength 28 Days

packing density method/ paste content	15%	20%	25%
cube compressive strength 28 days in N/mm2	31.513	35.88	42.7204
split tensile strength 28 days in N/mm2	21.71246	24.72132	29.43436

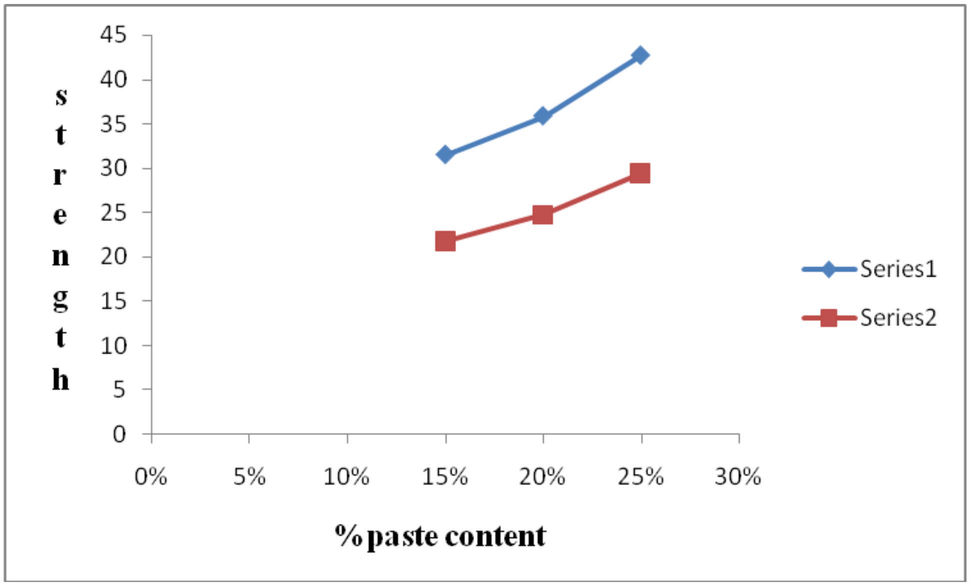

Figure 9 Comparison of Cube Compressive and Split Tensile Strength 28 Days

From the experimental results, the split tensile strength is found to be 0.689 times the cube compressive strength

CONCLUSIONS

From the present day work in the project based on the 7 days and 28 days cube compressive strength and the split tensile strength of M30 concrete grade the following conclusions can be drawn.

1. From the cube compressive strength obtained from various methods of the concrete mix design we came to a conclusion that packing density method is the most reliable and the most efficient method for designing high strength mixes with less cement content.
2. From the experiments we have drawn a correlation curve between paste content in excess of void content and the cube compressive strength which came to be linear with the equation $y=112.07x+14.29$ for 28 days strength and compared with the Raj et al (IOSR JMCE Volume 11, Issue2 Ver1 Mar-Apr, 2014, PP34-46) which got to be similar but, variation in the strength.

3. For solid suspension and compression packing models we have got a parabolic variation of strength with the excess paste content.
4. Although the solid suspension model is effective analytical for getting good strength using the equation provided but, it uses more cement content compared to packing density method.
5. Solid suspension model and compression packing models are showing the similar variation of 28 days and the 7 days cube compressive strength.
6. From the experiments on split tensile strength of 28 days of packing density method, we have got approximately 0.689 times the 28 days cube compressive strength of concrete.
7. From the experiments on the solid suspension model and the packing density mixes with 25% paste content in excess of void giving the M30 target mean strength.
8. Although the solid suspension model is giving good results for M30 but packing density method is effective with minimum cement content.

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