

# EFFECT OF LATERAL TORSIONAL BUCKLING ON WEB TAPERED I-BEAMS

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**ABSTRACT.** Lateral torsional buckling is considered as an ultimate limit state related to member buckling resistance. The buckling resistance is obtained by using a reduction factor. This reduction factor is the function of two other parameters, viz. the imperfection factor and the non-dimensional slenderness. The imperfection factor related parameter takes into account the initial member imperfection, residual stresses and other nonlinear effects. The non-dimensional slenderness depends upon the elastic critical moment ( $M_{cr}$ ) for lateral torsional buckling.  $M_{cr}$  can be calculated as per IS 800 (2007) (Annex-E, Clause 8.2.2.1). A comparative study between the values of  $M_{cr}$  and  $M_d$  (design bending strength) considering IS 800 (2007) code and AISC (1989) code is performed in the present paper. It indicates that the values of  $M_{cr}$  and  $M_d$  as per IS code are on the conservative side, as compared to the AISC code. It is therefore observed that there is a need for development of design formulae for tapered structural members in IS 800 (2007), considering the effect of web tapering ratio on the lateral torsional buckling of web tapered I-beams. Hence, it is essential to find out a factor in the  $M_{cr}$  equation, which will consider the interactions of the above parameters. The influence of the taper ratio on the critical load of web-tapered I-beams is seen to be very significant and must be taken into account when designing such members against buckling.

**Keywords:** Buckling, Lateral Torsional Buckling, Taper Ratio, Web-tapered Beams.

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## INTRODUCTION

The flexural capacity of beams with large unbraced lengths i.e. unrestrained beams is often limited by the mode of failure, known as Lateral Torsional Buckling (LTB). A beam is considered to be unrestrained when its compression flange is free to displace laterally and rotate. When an applied load causes both - lateral displacement and twisting of a member, lateral torsional buckling will occur. Figure 1 shows the lateral displacement and twisting experienced by a beam when LTB occurs. The applied vertical load results in compression and tension in the section flanges. The compression flange tries to deflect laterally away from its original position; whereas the tension flange tries to keep the member straight. The lateral bending of the section creates restoring forces that oppose the movement, because the tendency of the section is to remain straight. These restoring forces are not large enough to prevent the section from deflecting laterally, but together with the lateral component of the tensile forces, they determine the buckling resistance of the beam.

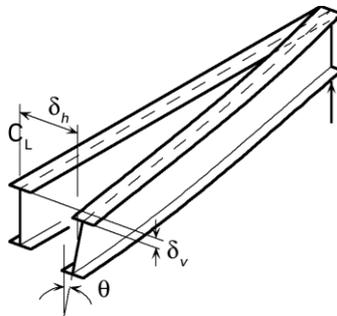


Figure 1 Lateral Displacement and Twisting (NSC 2006) [3]

In addition to the lateral movement of the section, the forces within the flanges cause the section to twist about its longitudinal axis. The twisting is resisted by the torsional stiffness of the section, which is dominated by the flange thickness. That is why a section with thicker flanges has a larger bending strength than that of the same depth section, with thinner flanges. LTB can be avoided by properly spaced and designed lateral bracings. The other factors affecting LTB are the proportions of beam cross sectional dimensions, material properties such as modulus of elasticity and shear modulus, length of the beam, section slenderness, support conditions, initial geometry imperfections, and the type and application of loading.

The effect of a destabilizing load is considered by the use of effective length given in Table 15 of IS 800 (2007), where the effective lengths are longer for destabilizing loads, as compared to those of the non-destabilizing loads. Factors  $C_1$ ,  $C_2$  and  $C_3$  are included to allow for the effect of different bending moment distributions and end restrained conditions.

In practice, beams are laterally braced in a variety of ways, in order to increase their buckling strength. Determining the brace force requirements for a system generally requires a large displacement analysis of an imperfect system. Most bracing studies usually focus on determining the maximum brace forces that are likely to occur in typical applications. This is particularly true for beam bracing in which the brace location and distribution of the loading can have a significant effect on the brace forces (Gill and Yura 1999) [4].

Over the past three decades, construction of buildings with frames comprising of web-tapered I-beams, manufactured from high tensile steel has become a standard practice. Their cross-

sectional profiles are intended to match the flexural strength close to the bending moment diagram, so that the requirement of the cross sections is well optimized. Despite several advantages of tapered structural members, they lack the appropriate simple and accurate design formulae in most of the codes of practice. The design solutions for tapered structural members are limited, because the available approaches consist of elastic design formulae, where taper effects are not properly accounted for. The stability of loaded tapered rafters was investigated by earlier researchers with proposals to design tapered rafters as uniform prismatic members, using additional factors.

$M_{cr}$  can be calculated as per IS 800 (2007) Annex-E, Clause 8.2.2.1. Here,  $C_1$ ,  $C_2$  and  $C_3$  factors depend upon the loading and end restrained conditions. When the loading is not due to a single central point load or due to full-length uniformly distributed load (udl), the published values IS 800 for  $C_1$  and  $C_3$  may be inaccurate and in some cases non-conservative. In addition, the  $C_3$  factor is only required for asymmetric sections. A comparative study as shown in Table 1 between the values of  $M_{cr}$  and  $M_d$  considering IS 800 (2007) code and AISC code indicates that the values of  $M_{cr}$  and  $M_d$  as per IS code are on the conservative side as compared to the AISC code.

It is therefore observed that there is a strong need for the development of design formulae in IS 800 (2007), for tapered structural members, considering the effect of geometric imperfections, effect of transverse loading applied at different heights with respect to the mid-height of the cross section and; effect of web tapering ratio, on the LTB of web tapered beams. Hence, it is essential to find out a factor in the  $M_{cr}$  equation, which will consider the interactions of these parameters. With the inclusion of this factor, it will be ensured that the material is used to its fullest capacity by optimizing the steel quantity, considering all the influencing parameters and thus reducing the steel consumption and saving the natural resources for future.

## REVIEW OF BEAM DESIGN AS PER IS 800 (2007) SPECIFICATIONS

Lateral torsional buckling is considered as an ultimate limit state related to the member buckling resistance. The buckling resistance is obtained by multiplying the resistance of the cross section by a reduction factor  $\chi_{LT}$ . This reduction factor is the function of two other parameters; the imperfection factor  $\alpha_{LT}$  and the non-dimensional slenderness  $\lambda_{LT}$ . The parameter  $\alpha_{LT}$  takes into account the initial member imperfection, residual stresses and other nonlinear effects. The non-dimensional slenderness  $\lambda_{LT}$  depends upon the elastic critical moments for lateral torsional buckling.

The elastic critical moment may be calculated from Equation (1) of Eurocode 3, derived from the buckling theory:

$$M_{cr} = C_1 \frac{\pi^2 E I_y}{(L_{LT})^2} \left\{ \left[ \sqrt{\left(\frac{K}{K_w}\right)^2 \frac{I_w}{I_y} + \frac{G I_t (L_{LT})^2}{\pi^2 E I_y} + (C_2 y_g - C_3 y_j)^2} \right] - (C_2 y_g - C_3 y_j) \right\} \quad (1)$$

Table 1 Values of  $M_{cr}$  and  $M_d$  using IS 800 (2007) and AISC code

Member Description	Member Size (mm)	Section type	IS 800 (2007) Clause 8.2.2.1		IS 800(2007) ANNEX-E		AISC-LRFD		
			$M_{cr}$ (kNm)	$M_d$ (kNm)	$M_{cr}$ (kNm)	$M_d$ (kNm)	Section type	$M_{cr}$ (kNm)	$M_d$ (kNm)
Web ( $d_w \times t_w$ )	152.4 × 2.54	Plastic	6.64	4.98	6.36	4.80	Compact	6.64	5.97
Flange ( $b_f \times t_f$ )	101.6 × 6.35	Compact					Compact		
Web ( $d_w \times t_w$ )	152.4 × 2.54	Plastic	81.68	32.57	63.06	29.04	Compact	49.17	44.25
Flange ( $b_f \times t_f$ )	304.8 × 6.35	Slender					Slender		
Web ( $d_w \times t_w$ )	152.4 × 6.35	Plastic	56.24	36.23	55.92	36.07	Compact	56.27	50.64
Flange ( $b_f \times t_f$ )	101.6 × 19.05	Plastic					Compact		
Web ( $d_w \times t_w$ )	152.4 × 2.54	Plastic	531.86	207.93	504.73	203.69	Compact	308.27	277.44
Flange ( $b_f \times t_f$ )	304.8 × 19.05	Compact					Compact		
Web ( $d_w \times t_w$ )	304.8 × 2.54	Slender	20.43	15.17	15.98	12.17	Non compact	20.43	18.38
Flange ( $b_f \times t_f$ )	152.4 × 6.35	Slender					Non compact		

In Equation (1),  $L$  is the beam length between points which have lateral restraint. For simply supported beams with intermediate lateral restraint, the effective length  $L_{LT}$ , as per the code is 1.2 times the length of the relevant segment in between the lateral restraints. This is how the code takes into account the destabilizing effect of the top flange loading.  $y_g$  is the distance between the points of load application and the shear centre.

When a beam buckles and twists, the displacement of the flanges, combined with the longitudinal bending stresses (compression and tension) set up a torque. The term  $y_j$  represents the coordinate of the centre about which the two torques rotate. This leads to  $y_j$  equal to zero for symmetric sections. For asymmetric sections  $y_j$  will be positive when the larger flange is in compression and negative when the smaller flange is in compression.  $K$  and  $K_w$  are effective length factors of the unsupported length accounting for boundary conditions at the end lateral supports. The effective length factors  $K$  and  $K_w$  vary from 0.5 for full fixity (against warping) to 1.0 for free (to warp) case and; 0.7 for the case of one end fixed and the other end free.

$C_1$  is equivalent uniform moment factor and;  $C_2$  and  $C_3$  are the factors which depend upon the loading and end restrained conditions. The values of  $C_1$ ,  $C_2$  and  $C_3$  are given in the Table 42 of IS 800 (2007). The factors  $C_1$  and  $C_3$  are little trickier, as they are not deterministic and most of them are quoted as values, formulae or graphs. When the loading is not due to a

single central point load or due to a full length udl, the published values of IS 800 (2007) for  $C_1$  and  $C_3$  can be inaccurate and in some cases non-conservative. Also the  $C_3$  factor is required only for asymmetric sections.

## NUMERICAL SIMULATIONS

In this paper, the effect of taper ratio and cross-sectional geometry on the stability of steel members that are subjected to bending is investigated. The effect of shear forces is considered negligible. The problem is studied by focusing on web-tapered I-beams with built-up cross-sections that are usually required in majority of the steel structures. The beams are considered simply supported in bending, while other boundary conditions can be easily dealt, with the approach proposed herein

### Nonlinear Finite Element Modeling

To investigate the lateral torsional buckling of beams, nonlinear finite element analysis is performed using the commercial software package, ANSYS R18.

### Material Properties

The properties of steel considered are: Young's modulus ( $E$ ) =  $2 \times 10^5$  MPa and Poisson's ratio ( $\nu$ ) = 0.3.

### Geometric Nonlinearities

Due to large deformations, the changing geometric configuration can cause the structure to respond nonlinearly. Hence geometric nonlinearity is considered in the current scenario.

### Finite Element Type and Mesh

A four-node structural shell element (SHELL181) from ANSYS R18 library has been used in the nonlinear finite element analysis, to investigate the lateral torsional buckling. The element has six degrees of freedom at each node; 3 translations in the x, y and z directions and 3 rotations about the x, y and z axis. The element is suitable for large rotation, large strain nonlinear application and load stiffness effect of distributed pressures. The performance of nonlinear buckling finite element convergence shows that the element mesh size of 25 mm (shown in Figure 2) is suitable and sufficient to apply in this investigation.

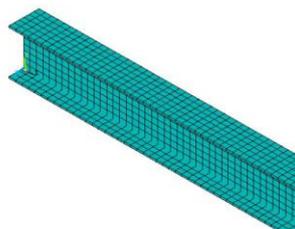


Figure 2 Shell Element Mesh Pattern

## Boundary Conditions and Load Application

In order to find the effect of tapering ratio of tapered I-shaped beams on the buckling analysis, loading and restraint conditions for single span tapered I-beams is considered. Point load is applied at the centre of the beam. Two typical symmetric tapered beams are considered in the study. The cross-sectional dimensions at the small end are given in Table 2. The two sections are identical to the sections originally used by Lee et al. (1972) [5] and M. L. Morrell & Lee (1974) [6]. The beam end conditions considered are both ends pinned.

Table 2 Section Dimensions

Dimension (mm)	Section- I	Section- II
$d_0$	152.4	152.4
$b$	101.6	101.6
$t_f$	6.35	19.05
$t_w$	2.54	6.35

Four different lengths of beam viz. 1.735 m, 2.275 m, 2.850 m and 3.415 m are considered in each case. Since the main objective of this investigation is to study the lateral torsional buckling of tapered beams, the cross sections are selected to have height to thickness ratio sufficient, to prevent the local buckling. The taper ratio  $\gamma = [(d_L - d_0)/d_0]$ , (where  $d_L$  and  $d_0$  are web depth at the large and small end of taper beam respectively) is varied as 0.0, 0.75, 1.5 and 3.0 which covers majority of the practical situations. These results are compared with the  $M_{cr}$  equation of IS 800 (2007), as per clause 8.2.2 and also as per Annex E.

## RESULTS AND DISCUSSION

- With reference to Figure 3, Figure 4 and Figure 5 for Section-I, it is observed that as the taper ratio increases from 0.0 to 3.0, there is drop in the nonlinear moment capacity of the tapered beam up to 65%. Whereas, as per IS 800 (2007), it is observed that as the taper ratio increases from 0.0 to 3.0 there is increase in the critical moment capacity of the beam by 80% (as per clause Annex E) and 100% (as per clause 8.2.2).
- With reference to Figure 6, Figure 7 and Figure 8 for Section-II, it is observed that as the taper ratio increases from 0.00 to 3.00, there is a drop in the nonlinear moment capacity of the tapered beam up to 42%. Whereas, as per IS 800 (2007), it is observed that as the taper ratio increases from 0.00 to 3.00 there is increase in the critical moment capacity of the beam by 14% as per clause Annex E and 39% as per clause 8.2.2
- The drop in the nonlinear moment capacity is more dominant in Section type I as against of Section type II. That is as for section having thinner webs, there is more drop as compared to the thicker web. As the length of the beam reduces, the drop in

the nonlinear moment capacity increases in both the type of sections i.e. Section-I and Section II. However, there is increase in the beam capacity as per IS800 (2007).

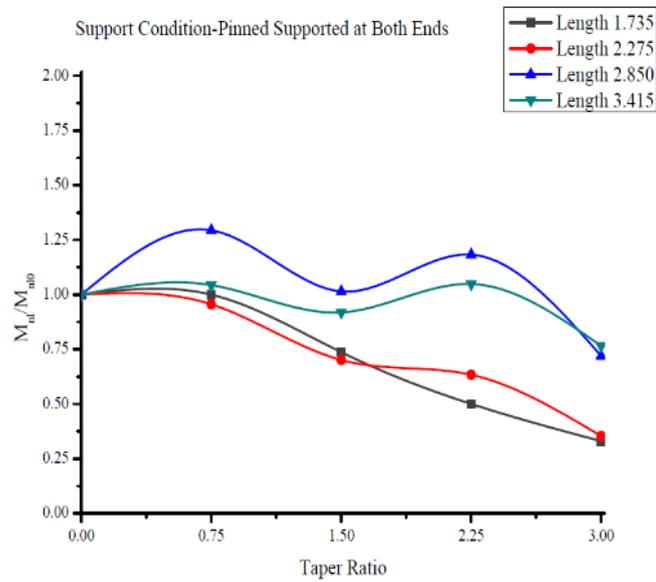


Figure 3 Nonlinear Moment Ratio ( $M_{ni}/M_{ni0}$ ) vs. Taper Ratio  $\gamma$  for Various Lengths of Tapered Beams with Concentrated Load applied at the Center of Top Flange (Section- I)

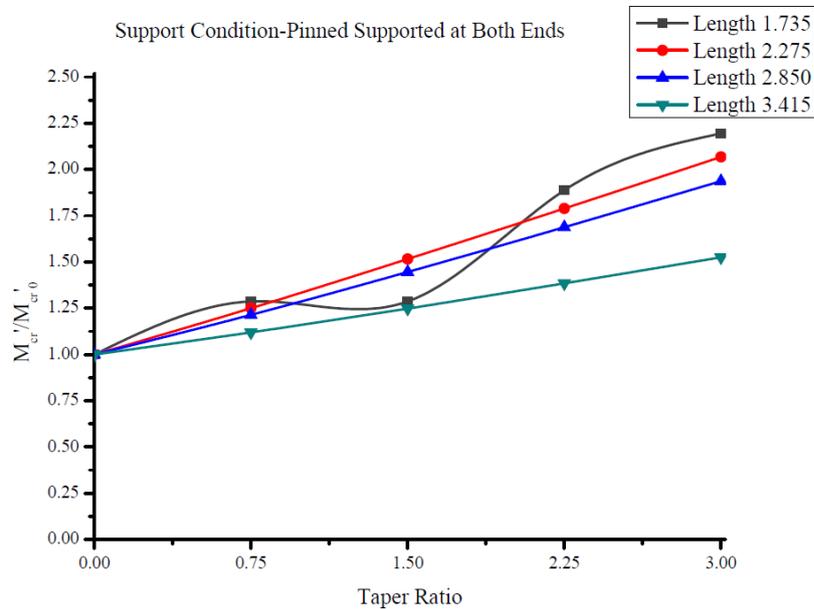


Figure 4 Elastic Critical Moment Ratio ( $M_{cr}'/M_{cr}'_0$ ) (As Per IS800 (2007) Clause 8.2.2) vs. Taper Ratio  $\gamma$  for Various Lengths of Tapered Beams with Concentrated Load applied at the Center of Top Flange (Section- I)

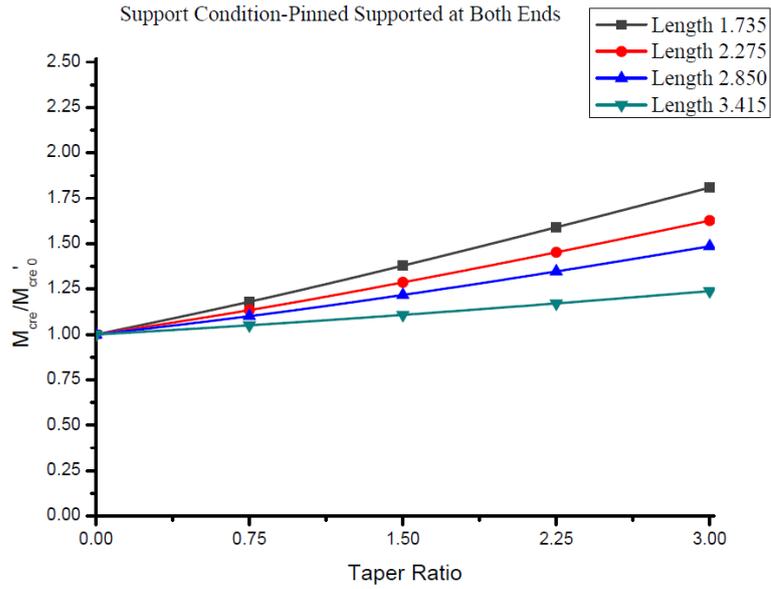


Figure 5 Elastic Critical Moment Ratio ( $M_{cre}/M_{cre'0}$ ) (As Per IS 800 (2007) Annex-E) vs. Taper Ratio  $\gamma$  for Various Lengths of Tapered Beams with Concentrated Load Applied at the Center of Top Flange (Section- I)

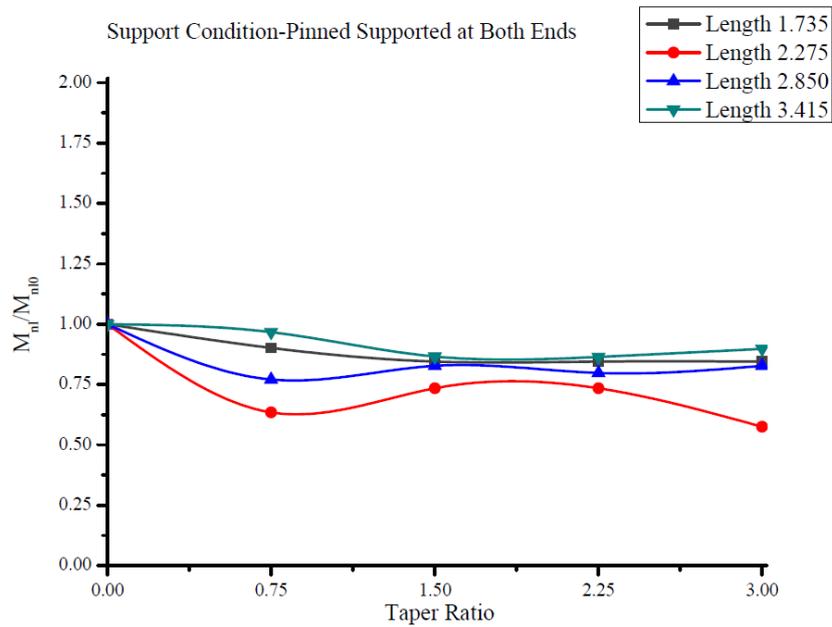


Figure 6 Nonlinear Moment Ratio ( $M_{nl}/M_{nl0}$ ) vs. Taper Ratio  $\gamma$  for Various Lengths of Tapered Beams with Concentrated Load Applied at the Center of Top Flange (Section- II)

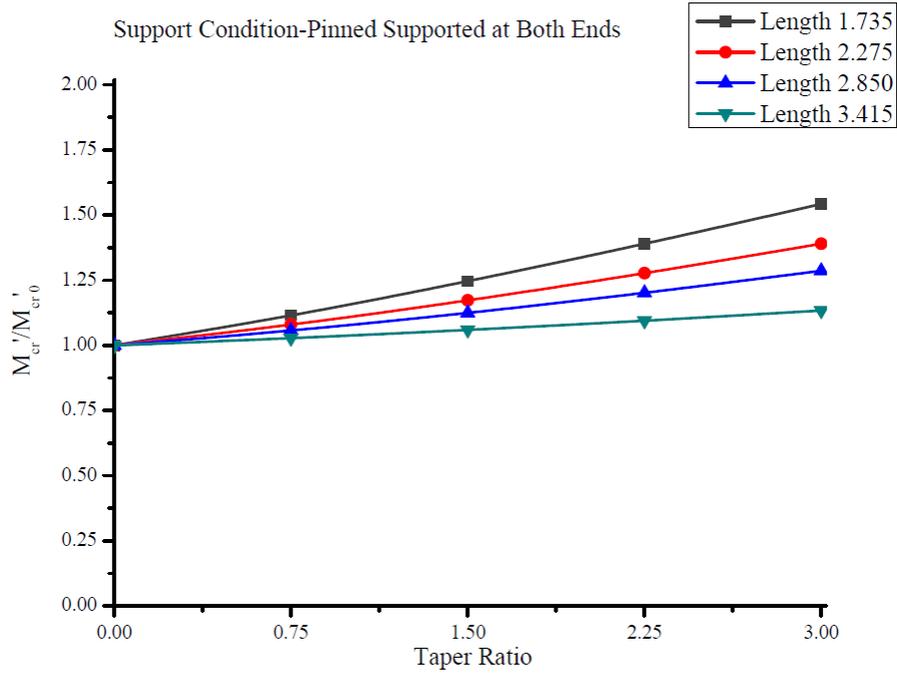


Figure 7 Elastic Critical Moment Ratio ( $M_{cr}'/M_{cr}'_0$ ) (As Per IS800 (2007) Clause 8.2.2) vs. Taper Ratio  $\gamma$  for Various Lengths of Tapered Beams with Concentrated Load Applied at the Center of Top Flange (Section- II)

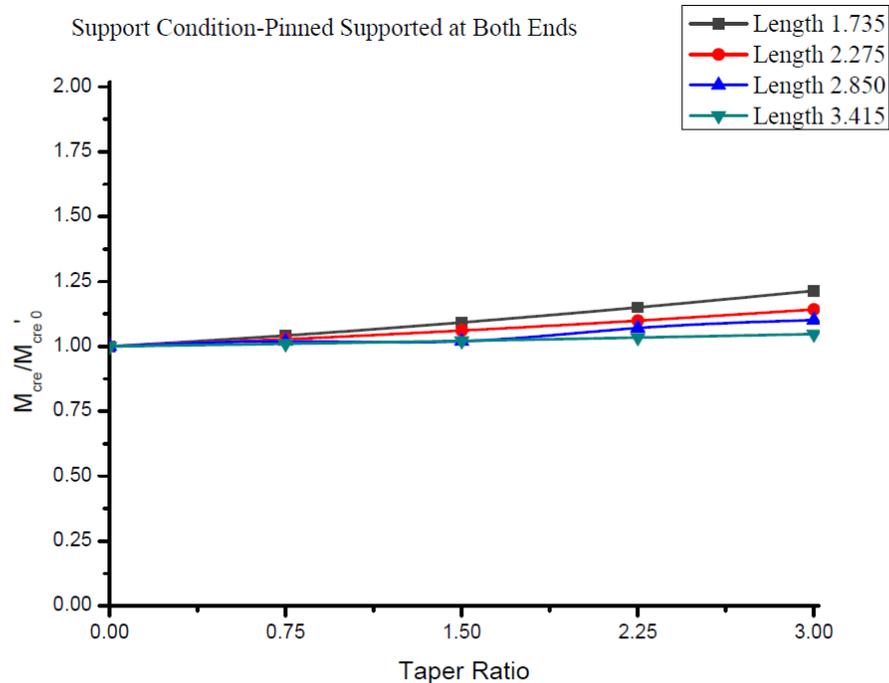


Figure 8 Elastic Critical Moment Ratio ( $M_{cre}'/M_{cre}'_0$ ) (As Per IS 800 (2007) Annex-E) vs. Taper Ratio  $\gamma$  for Various Lengths of Tapered Beams with Concentrated Load applied at the Center of Top Flange (Section- II)

## CONCLUSIONS

From the numerical study performed, it is observed that as the taper ratio increases from 0.00 (uniform beam) to 3.00 (tapered beam), there is a drop-in value of the critical load and critical moment. The influence of the taper ratio on the critical load of web-tapered I-beams is proven to be very significant and must be taken into account, when designing such members against buckling. Thus, effect of web tapering ratio on the LTB of web tapered beams needs consideration and hence, it is essential to find out a factor in the  $M_{cr}$  equation, which will consider the interactions of these parameters. With the inclusion of this factor, it will be ensured that the material is used to its fullest capacity by optimizing the steel quantity, considering all the influencing parameters and thus reducing the steel consumption and saving the natural resources for future.

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