PREDICTION OF MEAN AND DESIGN FATIGUE LIVES OF PLAIN AND FIBRE REINFORCED SELF COMPACTING CONCRETE BEAMS IN FLEXURE

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ABSTRACT. The mean and design fatigue lives of the Plain and Fibre Reinforced Self Compacting Concrete (FRSCC) containing Steel Fibres(SF) and Polypropylene Fibres (PPF) have been estimated in this paper. The plain SCC and FRSCC with 100% PPF, 25% PPF + 75% SF and 100%PPF at 1% total fibre volume fraction have been used in this investigation. Approximately 200 flexural fatigue tests and 72 static fatigue tests have been conducted on the beam specimen of 100 x 100 x 500 mm size. The Weibull parameters have been calculated using the graphical method and method of moment, which are subsequently used to predict the mean and design lives of FRSCC mixes. The flexural fatigue tests were conducted at different stress levels ranging from 0.65 to 0.9. The performance of FRSCC mix containing 100%PPF have been found to be better in terms of reduced variability and mix having 100%SF have better mean and design fatigue lives as compared to other mixes.

Keywords: Hybrid Fibre Reinforced Self Compacting Concrete, Flexural Fatigue, Mean and Design fatigue lives. Weibull parameters.

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INTRODUCTION

Despite early interest in fatigue in metals, interest in non-metallics, particularly plain concrete and Steel Fibre Reinforced Concrete (SFRC), lagged behind for many years. Concrete fatigue investigations, like those on metals, have also been motivated by practical problems. With the development of highway system, there has been keen interest in fatigue behaviour of concrete composites. Bridge decks, Airport pavements, off-shore structures are predominantly subjected to fatigue loading during their life. *ASTM STP-91-A (1963)* [1] defines fatigue as the process of progressive localized permanent structural change occurring in a material subjected to conditions which produce fluctuating stresses and strains at some points and which may culminate in cracks or complete fracture after a sufficient number of fluctuations. Very high repeated loadings caused due to earthquakes or any other natural disaster may cause a structure to fail in less than 100 cycles, which may be termed as low-cycle fatigue. However, low-cycle fatigue has been termed as approximately up to 1000 cycles of repeated loading and high-cycle fatigue as the range from 1000 to 10000000 cycles [2]. Highway and railway bridges, concrete rail road ties, and airport and highway pavements, may be considered under the high-cycle fatigue class. Concrete subjected to fatigue loading may have excessive cracking and can fail under a certain number of cycles of loads even if the maximum fatigue load is less than its static strength. Therefore, definition of fatigue strength is given as the fraction of the static load that concrete can sustain for a given number of cycles.

In spite of a large number of studies on Fibre Reinforced Self Compacting Concrete (FRSCC) under statically applied loads, only a few investigations are available on its fatigue characteristics and the data on the subject is scanty. Therefore, the present investigation was planned to carry out an in-depth study on the flexural fatigue characteristics of FRSCC containing steel fibres and polypropylene fibres.

The present investigation was planned to estimate the mean and design fatigue lives of SCC and FRSCC. The plain SCC and FRSCC with 100% PPF, 25% PPF + 75% SF and 100%PPF at 1% total fibre volume fraction have been used in this investigation. Approximately 200 flexural fatigue tests and 72 static fatigue tests have been conducted on the beam specimen of 100 x 100 x 500 mm size. As a prerequisite, the fatigue life distributions of FRSCC mixes have been first examined using two-parameter Weibull distribution by graphical method and method of moments. Using the average values of the distribution parameters, the mean and design fatigue lives for FRSCC have been estimated.

EXPERIMENTAL PROGRAMME

In the present study the Ordinary Portland Cement of 43 grade conforming to requirements of IS 8112-1989 has been used. Crushed stone aggregates (below 12.5 mm) of specific gravity 2.78 were used as coarse aggregates and locally available coarse sand of specific gravity 2.75 conforming to Zone II grain size distribution was used as fine aggregates. The water absorption of coarse aggregates was 0.28 % and that of fine aggregates was 0.25%. The Class F fly ash was used.

Glenium 51, a polycarboxylic ether based super-plasticizer as admixture was used in suitable dosages to obtain the required FRSCC mixes. Glenium Stream II was used as

ViscosityModifying Agent (VMA). The following type of fibres shall be used at 1% volume fraction.

- Corrugated Steel Fibres (SF) of 1mm dia. and 30mm length and specific gravity of 7.8.
- Polypropylene Fibres (PPF) of 0.7mm dia and 10mm length and specific gravity of 0.91.

Suitable SCC mixes with different proportions of steel and polypropylene fibres were obtained through trials as per EFNARC guidelines [3]. The Mix Proportions of the mix are shown in Table 1.

Dosage of Super-plasticizer (SP) was kept in range of 0.6–1.2% by weight of cement content for various FRSCC mixes and dosage for Viscosity Modifying Agent (VMA) was kept between 0.35-0.5 percent of the cement content in order to meet required EFNARC limits for SCC. For compressive strength tests the specimens used were 150 x 150 x 150 mm cubes whereas standard prisms of size 100 x 100 x 500 mm were used for the static flexural strength tests.

Table 1 Mix Proportions of the SCC

CEMENT	FLY ASH	FINE AGGREGATES	COARSE AGGREGATES	WATER
410	205	846	602	277

The specimens were cast in the different batches. Each batch had fourteen flexural test specimen and six cubes for testing the 28-day compressive strength of each mix. The specimen for compressive strength tests were cured for 28 days, whereas, the specimens for flexural strength tests were cured for 90 days. The compressive strength tests were conducted on concrete cubes in a 200 tonnes Universal Testing Machine. The flexural strength and flexural fatigue of FRSCC was obtained by testing prism specimens under four point bending test. All the flexural strength tests were done on a 100 kN servo-controlled actuator. The fatigue tests were performed at different stress levels of 0.7, 0.75, 0.8 and 0.85 for the mixes with different fibre combinations.

The quality of each batch was tested by its compressive strength at 28 days. The compressive strength of SCC is 28.9 MPa, for mix having 100% PPF is 29.45, for mix having 25%PPF+75%SF is 44.32 MPa and for 100%SF mix is 41.23 MPa. The specimens for flexural strength tests were cured for 90 days to avoid possible gain in strength during fatigue tests. The static flexural strength tests are prerequisite for estimating maximum and minimum load limits for flexural fatigue tests. The average static flexural strength was 4.63 MPa, 5.44 MPa, 8.12 MPa and 7.34 MPa for SCC and FRSCC mix having 100%PPF, 25%PPF-75%SF and 100%SF, respectively. After the static flexural strength of a particular batch was established, the flexural fatigue tests were conducted. Constant-amplitude non-reversed flexural loads were applied at a frequency of 10 Hz. The fatigue test was terminated as and when the specimen failed or a maximum limit of 2×10^6 load cycles was reached (whichever was earlier). The fatigue life of each specimen at a particular stress level was recorded as number of cycles to failure (N). In total, 200 flexural fatigue tests and 72 complementary static flexural tests were conducted in this investigation.

FATIGUE TEST RESULTS AND ANALYSIS

For each volume fraction of fibre in a particular FRSCC mix, the first test was conducted at the highest stress level and the numbers of cycles to failure were reported as fatigue life 'N'. Subsequent tests were conducted by lowering the stress levels in a systematic manner. Since the fatigue results of FRSCC exhibits a range of data with inherent scatter as opposed to single absolute value, a probabilistic approach is evidently needed for estimating the fatigue life of concrete. Probabilistic analysis of fatigue data at each stress level has been modelled by two parameter Weibull distribution in this study. The two-parameter Weibull distribution has increasing hazard function and thus is the most suitable for fatigue studies. The distribution of fatigue life of normally vibrated concrete as well as self compacting concrete has been shown to approximately follow the two-parameter Weibull distribution [1,4 – 7]. The parameters of the Weibull distribution have been calculated by two methods, the graphical method and the method of moments.

Graphical Method

Equation 1 represents a linear relationship between $\ln[\ln(1/L_N)]$ and $\ln(n)$ and is used to verify the two-parameter Weibull distribution for analysis of the fatigue life of concrete [4,8.9]. The survivorship function $L_N(n)$ of the two-parameter Weibull distribution may be written as

$$\ln\left[\ln\left(\frac{1}{L_N}\right)\right] = \alpha \ln(n) - \alpha \ln(u) \qquad (1)$$

Where n is the specific value of N (number of cycles to failure or fatigue life); α is the shape parameter at stress level S and u is the scale parameter at stress level S.

This equation represents a linear relationship between $\ln[\ln(1/L_N)]$ and $\ln(n)$ and is used to verify the two-parameter Weibull distribution. To obtain a plot between $\ln[\ln(1/L_N)]$ and $\ln(n)$, the fatigue test results for a given stress level are firstly arranged in ascending order of the number of cycles to failure and then the empirical survivorship function L_N is then obtained from the following expression [4,8,11]:

$$L_N = 1 - \frac{i}{k+1} \qquad (2)$$

Where, i = order number, and k = total number of fatigue data points at a given stress level S. It has been observed that at each stress level, the fatigue test result data follows approximately a linear trend on the graph, so it can be concluded that the Weibull distribution is a reasonable assumption for the statistical analysis of fatigue life data of FRSCC.

Method of Moments

In this method, the parameters are obtained by calculating the appropriate sample moments, such as sample mean and sample variance. The following expressions have been used to obtain the values of shape parameter α and scale parameter u [8]:

$$\alpha = (\mathrm{CV})^{-1.08} \tag{3}$$

$$u = \frac{\mu}{\mathrm{T}\left(\frac{1}{\alpha}+1\right)} \tag{4}$$

Where, CV = $\frac{\sigma}{\mu}$ is the coefficient of variation of the fatigue data sample at a particular stress level S, σ is the standard deviation and μ is the mean of the test data at particular stress level S. $\Gamma(\ )$ is a gamma function. The average parameters of the Weibull distribution for the fatigue life data of FRSCC at stress levels of 0.7, 0.75, 0.8 and 0.85 obtained by taking average of two methods has been presented in Table-2.

Table 2 Average values of Weibull Parameters for Fatigue Life Data of FRSCC

STRESS LEVEL 'S'	PARAMETER	MIX DESIGNATION			
		SCC	100% PPF	25% PPF + 75%SF	100% SF
0.85	α	4.4869	3.0668	2.1148	1.8885
	u	2041	3957	96906	54365
0.8	α	3.7248	2.4281	1.8027	1.6928
	u	15063	32064	475289	229518
0.75	α	3.2136	2.0295	1.5831	1.5028
	u	65792	152727	1080308	1078329
0.7	α	2.8281	1.7844	1.4636	1.4147
	u	179787	364980	1379483	1175481

The preliminary results on FRSCC mixes indicate that the probability distribution of fatigue life at a given stress level S, can be approximately represented by the two-parameter Weibull distribution. It can be seen that the shape parameter increases with increase in stress level. Higher is the value of shape parameter, lesser is the variability in test results. It means the variability is more for lower stress level and vice versa. The value of shape parameter also increases with the increase in percentage of polypropylene fibres in the mix. So it can be concluded that polypropylene fibre inclusion decreases the variability in fatigue test data results and makes the concrete life prediction more reliable.

ESTIMATION OF MEAN FATIGUE LIVES

The calculation of mean and design lives is the primary objective of this investigation. The mean fatigue life E[N] can be obtained by the following equation [10]:

$$E[N] = C\,(S)^{m} \exp\left[\frac{0.5772}{\alpha}\right]\Gamma\left(1+\frac{1}{\alpha}\right) \tag{5}$$

Where, S is the stress level, m and C are empirical constants, E[N] is mean fatigue life, Γ( ) is the gamma function, α is the shape parameter of the Weibull distribution and N is the number of cycles to failure. The values of m and C are obtained using the following S–N relationship [10]:

$$N\,(S)^{m} = C \tag{6}$$

Taking log on both sides

$$\log_{10}(N) = \log_{10}C - m\log_{10}(S) \tag{7}$$

The Equation (7) can be written as

$$Y = a + bX \tag{8}$$

Where, $Y = \log_{10}(N)$, $X = \log_{10}(S)$, $a = \log_{10}C$ and $b = -m$

The coefficients a and b for FRSCC mixes can be obtained by fitting Equation 8 into the respective fatigue life data obtained in this experimental study. The analysis of the fatigue life data of SCC and FRSCC mixes containing 100%PPF, 25% PPF+75%SF and 100%SF using Equations 6, 7 and 8 are presented in Figures 1 – 4, respectively.

The values of the constants of this Equation 6, m and C are obtained from the regression analysis. The S–N relationship for the FRSCC mixes as depicted by Equation 6 can be represented in terms of the following relations:

For	SCC:	$N\,(S)^{18.18} = 179.89$	(9)
For FRSCC containing	100% PPF:	$N\,(S)^{19.25} = 262.42$	(10)
	25% PPF +75% SF:	$N\,(S)^{17.99} = 3258.37$	(11)
	100% SF:	$N\,(S)^{19.86} = 1428.89$	(12)

Using the average values of the Weibull parameters listed in Table 2, and the values of the coefficients m and C as given in Equations 9 – 12 can be used to calculate the mean fatigue lives for different FRSCC mixes. The calculated mean fatigue lives corresponding to different stress levels are presented in Table 3. The FRSCC mix having 25% PPF +75% SF has the highest mean fatigue lives.

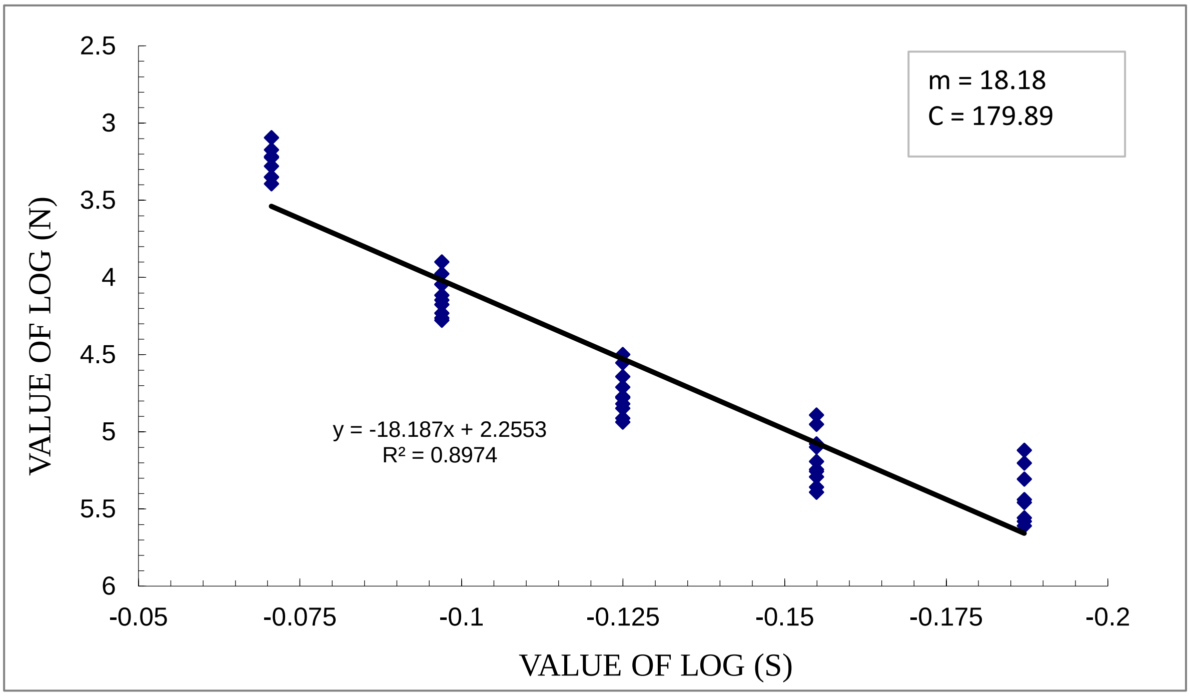

Figure 1 Estimation of Coefficients m and C of Eq.(6) for fatigue life data of SCC.

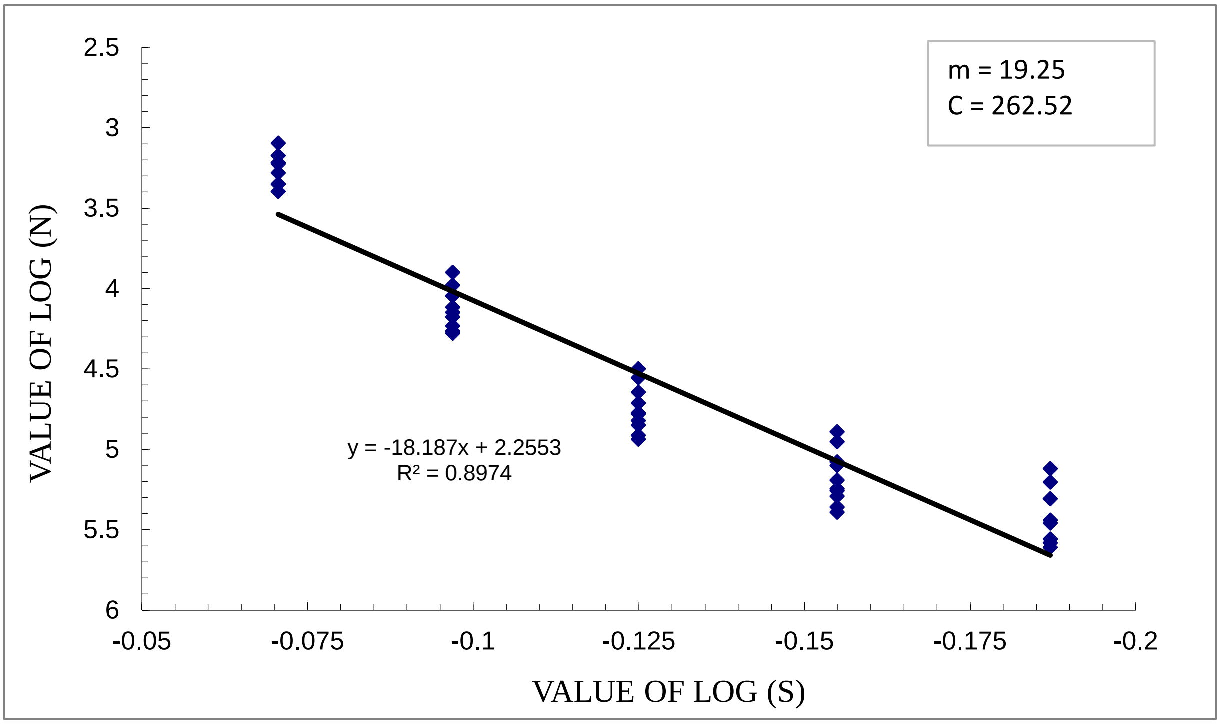

Figure 2 Estimation of Coefficients m and C of Eq.(6) for mean fatigue life data of FRSCC(100% PPF + 0% SF).

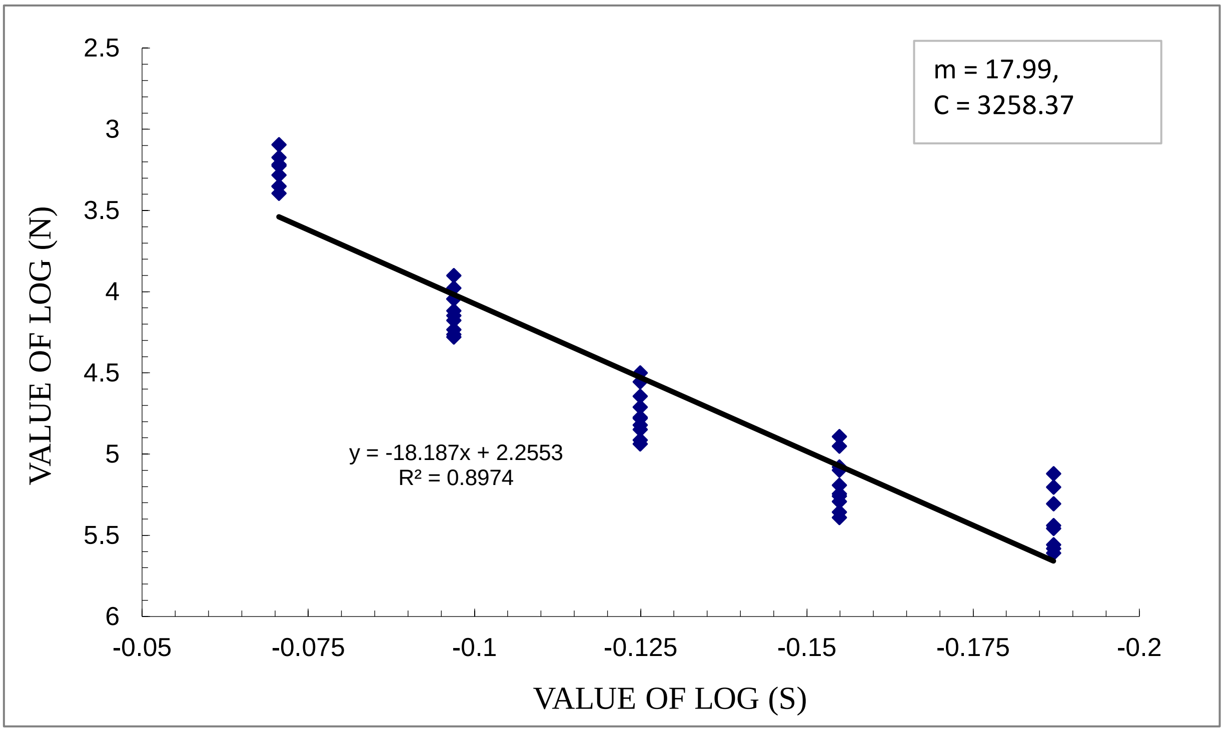

Figure 3 Estimation of Coefficients m and C of Eq.(6) for fatigue life data of FRSCC(25%PPF + 75%SF).

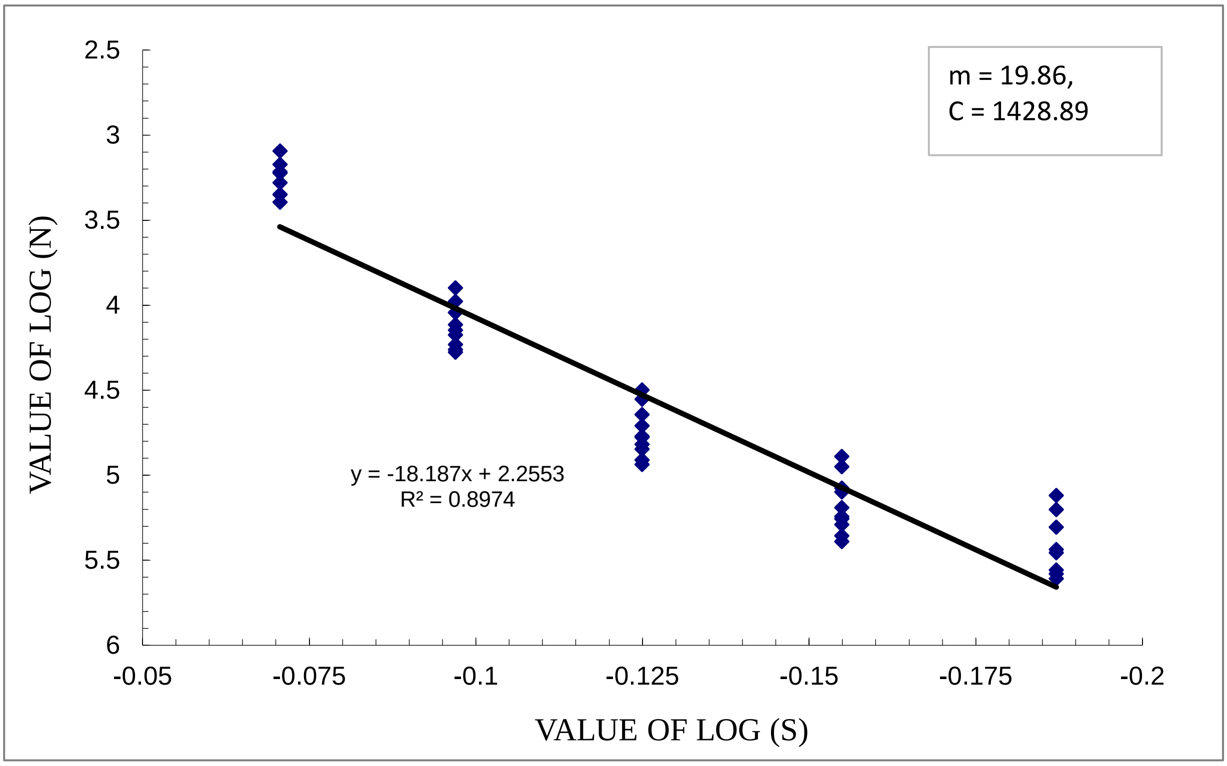

Figure 4 Estimation of Coefficients m and C of Eq.(6) for mean fatigue life data of FRSCC(0% PPF + 100% SF).

Table 3 Mean Fatigue lives of FRSCC mixes at different stress levels

STRESS LEVEL S	MEAN FATIGUE LIFE E[N]			
	SCC	100%PPF	25%PPF + 75%SF	100%SF
0.85	3883	7092	77221	47266
0.80	11955	23771	239919	162854
0.75	39468	85804	799389	612107
0.70	141272	335616	2849638	2469404

ESTIMATION OF DESIGN FATIGUE LIVES

Due to the nature of fatigue phenomenon and the uncertainties with the anticipated loads, the fatigue life prediction of concrete structures does not have a single value. But the design life can be calculated for different probabilities of failure at all stress levels within a stress spectrum. Then design engineer can chose the design life corresponding to chosen probability of failure as per the code provisions. The design fatigue life N_D should be selected such that there is only a small probability that a fatigue failure will occur. Firstly, the distribution function is determined, and then the design fatigue life may be selected corresponding to an acceptable reliability. The design reliability is expressed as $L_N = 1 - P_f$, in which P_f is the probability of failure. Thus the design fatigue life N_D corresponding to a permissible probability of failure P_f can be calculated using following equation:

$$N_D = u\left(ln\frac{1}{1-Pf}\right)^{1/\alpha} \qquad (13)$$

Using the average values of the Weibull parameters α and u at different stress levels for the fatigue life data of FRSCC, Equation 13 has been used to calculate the design fatigue lives corresponding to selected acceptable probabilities of failure (P_f), that is, 0.01, 0.05, 0.10, 0.15 and 0.25. The calculated design fatigue lives corresponding to the stress level and probabilities of failure are listed in Tables 4 – 7 for various mixes.

Table 4 Design fatigue lives of SCC at different Probabilities of Failure.

S	PROBABILITY OF FAILURE				
	0.01	0.05	0.1	0.15	0.25
0.85	732	1053	1236	1361	1546
0.8	4381	6786	8233	9249	10781
0.75	15722	26108	32663	37379	44648
0.7	35346	62898	81130	94566	115726

Table 5 Design fatigue lives of FRSCC (100%PPF + 0%SF) at different Probabilities of Failure

S	PROBABILITY OF FAILURE				
	0.01	0.05	0.1	0.15	0.25
0.85	944	1599	2017	2320	2790
0.8	4381	8591	11567	13836	17518
0.75	19244	42903	61130	75655	100191
0.7	29145	72487	108395	138111	190048

Table 6 Design fatigue lives of FRSCC (25%PPF + 75%SF) at different Probabilities of Failure

S	PROBABILITY OF FAILURE				
	0.01	0.05	0.1	0.15	0.25
0.85	10464	22657	31870	39137	51303
0.8	36035	88867	132393	168308	230914
0.75	53530	150019	236482	311030	446276
0.7	51700	157511	257619	346438	511840

Table 7 Design fatigue lives of FRSCC (0%PPF + 100%SF) at different Probabilities of Failure

S	PROBABILITY OF FAILURE				
	0.01	0.05	0.1	0.15	0.25
0.85	4750	11292	16554	20841	28229
0.8	16117	42252	64669	83559	117123
0.75	46023	135787	218961	291950	426514
0.7	44420	140874	234528	318771	477634

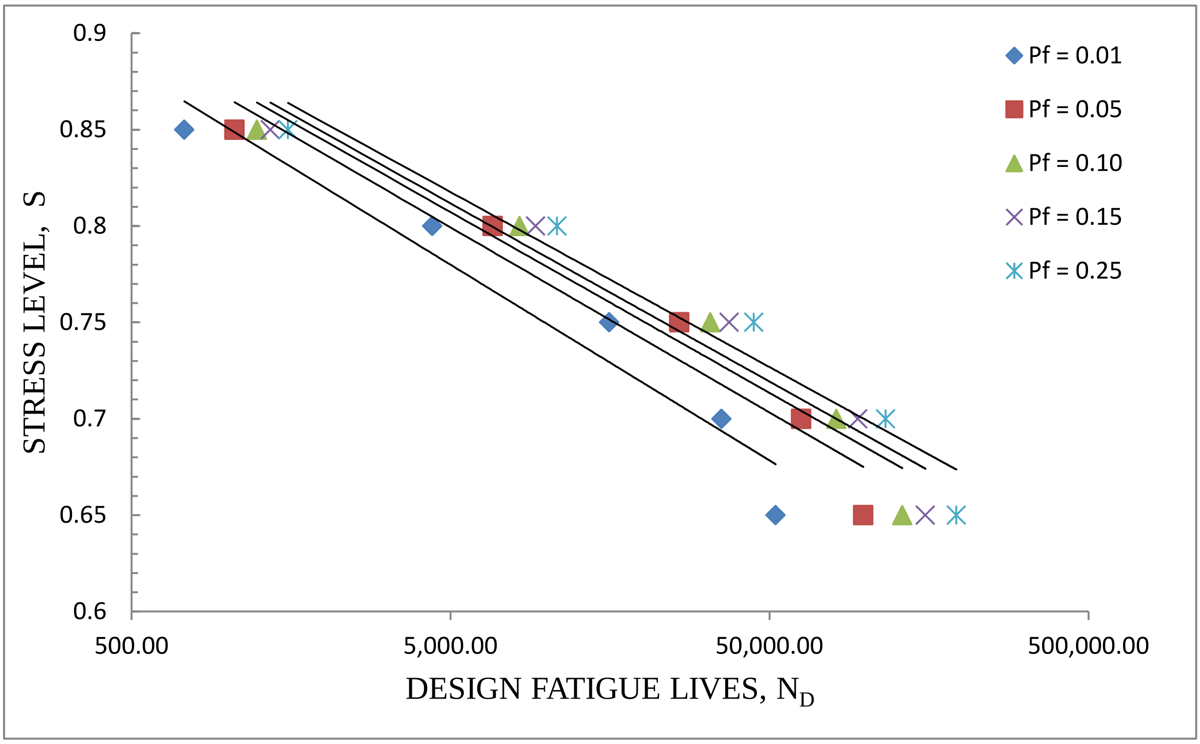

Figure 5 Design fatigue curves of SCC at different probabilities of failure

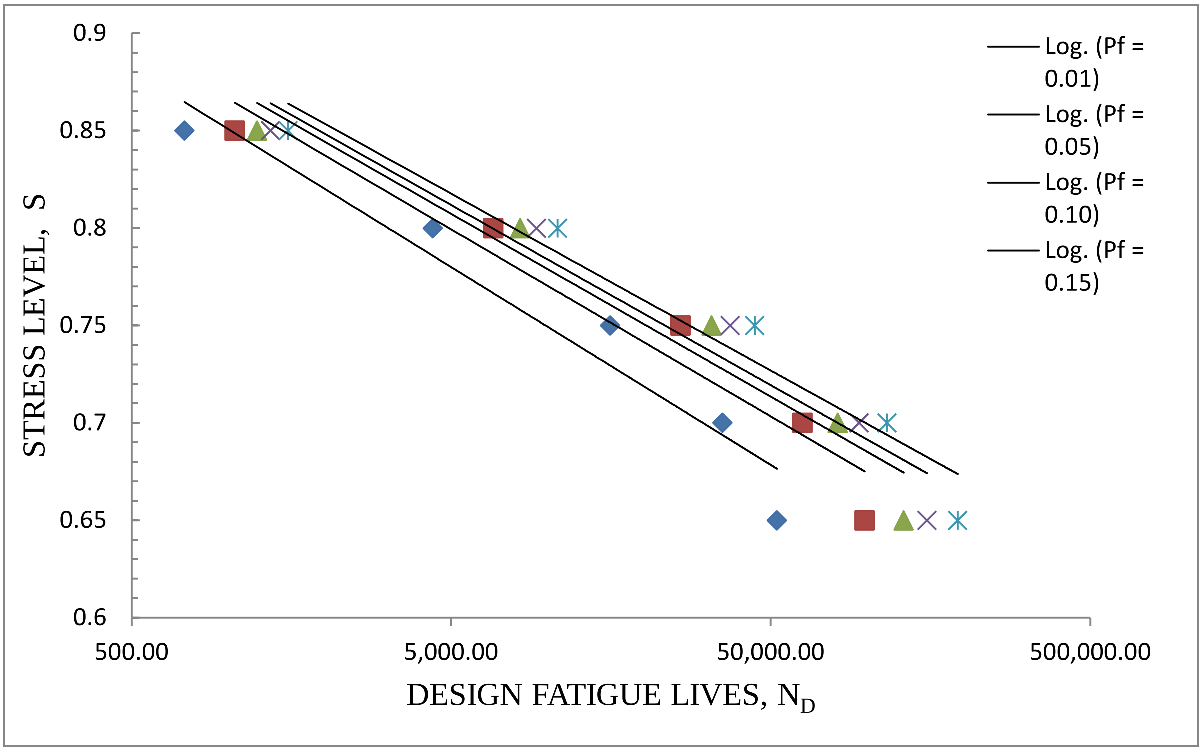

Figure 6 Design fatigue curves of FRSCC (100%PPF + 0%SF) at different probabilities of failure

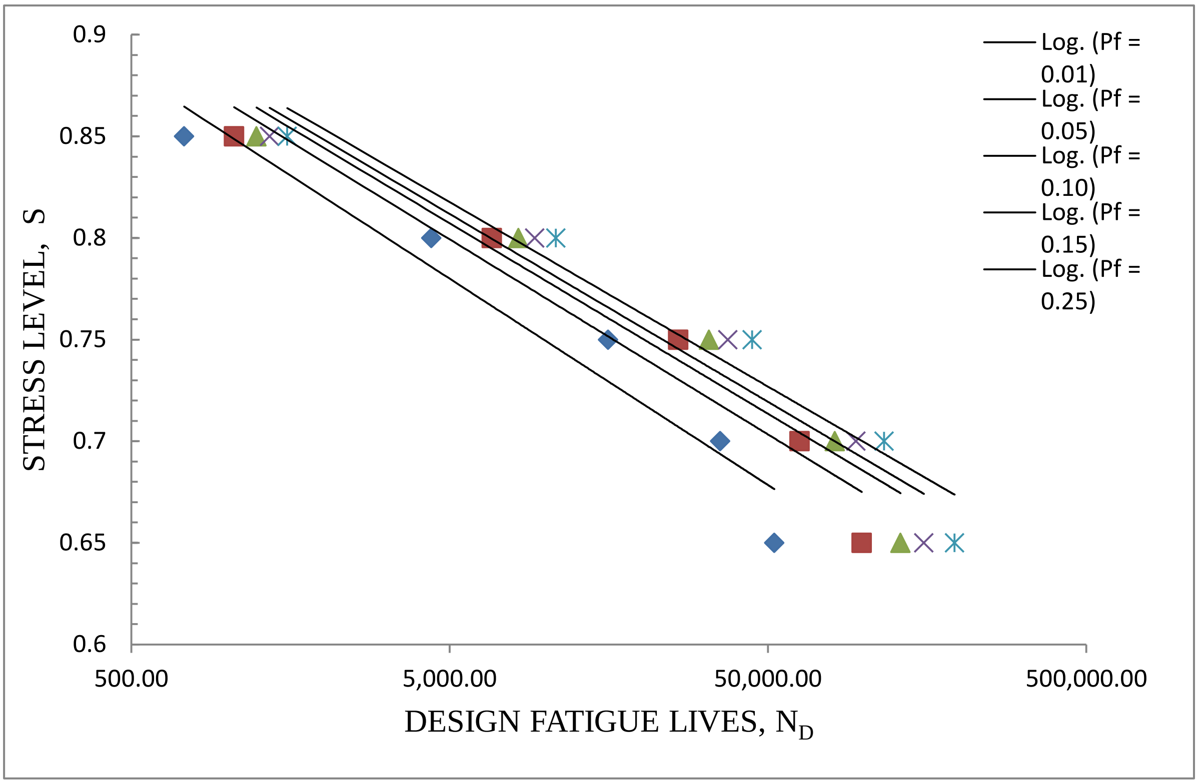

Figure 7 Design fatigue curves of FRSCC (25%PPF + 75%SF) at different probabilities of failure

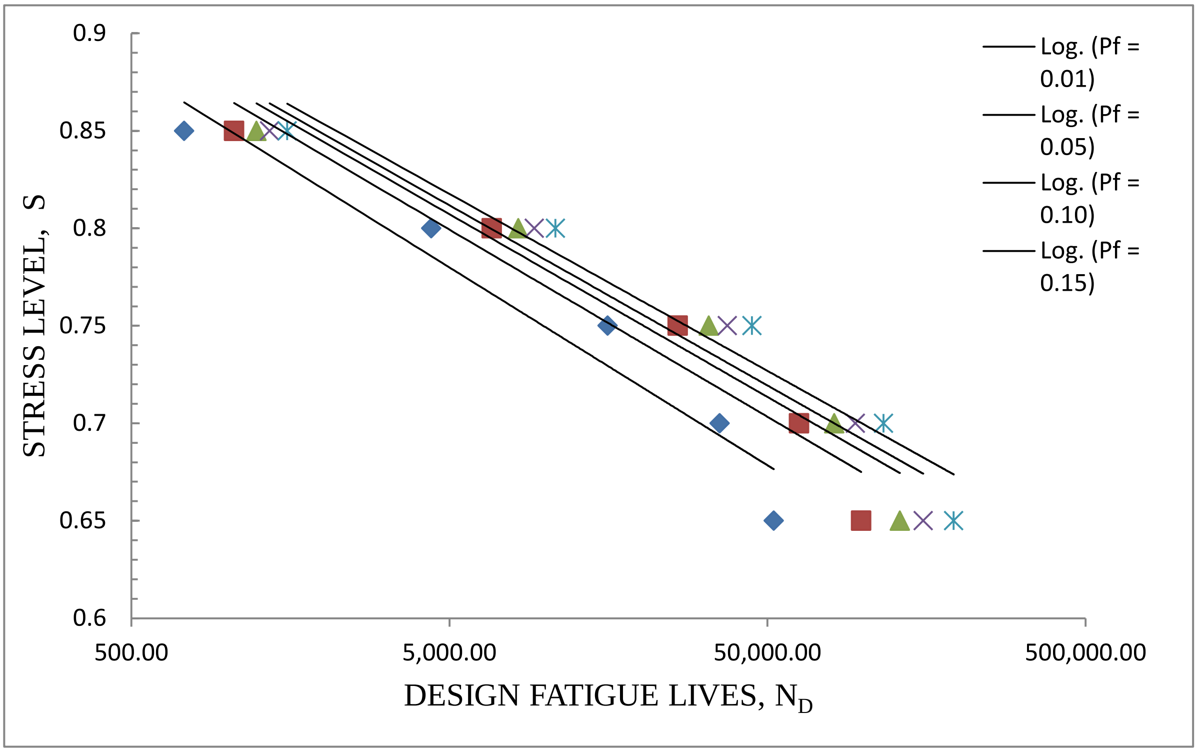

Figure 8 Design fatigue curves of FRSCC (0%PPF + 100%SF) at different probabilities of failure

The "design fatigue-life curves" have been generated using the design fatigue lives and are shown in Figures 5 – 8, which may be used by the design engineers. It has been observed that for a particular stress level *S*, the FRSCC with 25%PPF +75% SF has the highest design fatigue-life. The mix with 100%PPF has the least fatigue lives amongst these mixes. This indicates that the best fatigue performance is given by FRSCC with 25%PPF+75%SF.

CONCLUSION

In this study, experiments have been performed to estimate the flexural fatigue lives for FRSCC mixes at different stress levels. A probabilistic approach has been used to establish the probability distributions of FRSCC using the two parameter Weibull distribution. The FRSCC mix having 100% PPF has the maximum value of shape parameter. This mix can be adjudged as the best performer in terms of variability in fatigue data and hence is the most reliable mix.

Further, the fatigue strength prediction using *S-N* relationship has been done. The equations and curves for mean fatigue lives of FRSCC mixes containing different combinations of steel and polypropylene fibres have been developed. These curves can be used to predict the flexural fatigue strength of FRSCC. Design fatigue lives have also been determined for all FRSCC mixes and design fatigue life curves have been generated. It has been concluded that for a particular stress level *S*, the FRSCC with 25%PPF+75% SF has the highest mean and design fatigue life.

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