STUDY ON THE BEHAVIOUR OF CONCRETE BEAMS UNDER DIFFERENT LOADING CONDITIONS

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ABSTRACT. The paper is concerned with the behaviour of concrete beams under different loading conditions. Method of Initial Functions (MIF) has been used to study the effect of different types of loading on the behaviour of concrete beams. It is an analytical method based on elasticity theory. This method gives exact solution of problems without using assumptions about physical conditions of beams. A deep concrete beam is analysed for four different types of loading having same intensity. Different loadings are uniformly distributed load, sinusoidal load, triangular load and uniformly varying load. Ends of beam are taken as simply supported.

Keywords: Method of Initial functions, Deep beam, Displacement, Stress, Loadings.

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INTRODUCTION

The results obtained by bending theory are far from the actual physical behavior. Beam theories which are based on assumptions produces errors in stresses and deflections. So, there is a need of theory capable of analyzing beams exactly. In this paper method of initial functions (MIF) is used for the analysis of beams for different types of loadings. It gives exact solutions for variants of quandaries without the utilization of postulations about the character of stress and strain. In comparison to Bernoulli's beam theory and Timoshenko beam theory, generally used for analysis of beams, this method requires no assumptions regarding the behavior of beams.

Method of initial functions (MIF) was developed for the analysis of plates and shells. In this method, partial differential equations of stress and deflections are expanded in Maclaurin's series in the thickness coordinate. The solutions are obtained in terms of unknown initial functions on the reference plane (Vlasov 1957).Two dimensional elasticity equations were used in this method (Timoshenko and Goodier 1951). It is used for the analysis of beams under symmetric central loading and uniform loading for different end conditions (Iyengar et al., 1974). MIF had been applied for deriving theories for laminated composite thick rectangular plates. The governing equations had been obtained for perfectly and imperfectly bonded plates subjected to normal loads (Iyengar and Pandya, 1986). Governing equations were developed for composite laminated deep beams by utilizing method of initial functions and results were compared with the available theory (Dubey 2000). It was successfully applied for the analysis of brick filled reinforced concrete beams (Patel et al., 2013). It was used to obtain results for three different cases of depth-span ratios and was compared with bending theory and finite element method (Patel et al., 2014a). Effect of elastic properties on the behaviour of beams was studied (Patel et al., 2014b).

In the case of laminated beams (Silverman 1980) it is quite difficult to assume a distribution of stresses and deflection with a fair amount of accuracy. To study the behavior of composite sandwich structures and develop simple models to explain this behavior as a function of material, geometric and loading parameters, Du et al., (2016) developed a method to convert the cross sectional areas of unbonded prestressed tendons to the equivalent cross sectional area of non-prestressed steel. The computed deflections are compared with some available experimental results, including beams with external unbonded steel tendons and those with external unbonded aramid fibre reinforced polymer tendons. This method gives satisfactory predictions of deflection till the yielding of non-prestressed steel. Li and Li, (2016) studied three-point bending of a beam based on the Timoshenko beam theory. Large deflection and large rotation of a beam resting on simple supports with friction are calculated for concentrated force acting at the midspan. Using the Lagrangian kinematic relations, a system of non-linear differential equations are obtained for a prismatic shear-deformable Timoshenko beam. Solutions are derived analytically for deflection, horizontal displacement and rotation of cross-section and are compared with the classical Timoshenko beam theory.

MIF FORMULATION

In this method the stress σ_x, is eliminated first from the equations of equilibrium and the stress-displacement relations of elasticity theory. After elimination four equations are obtained, two in terms of displacements and two in terms of stress components. Solutions of these four equations are taken in the form of exponential series in the thickness coordinate and involving functions and their derivatives on a specified initial plane.

The equations of equilibrium for solids without body forces for two-dimensional case are:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \tag{1}$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0 \tag{2}$$

The stress-strain relations for isotropic material are:

$$\sigma_x = C'_{11}\varepsilon_x + C'_{12}\varepsilon_y \tag{3}$$

$$\sigma_y = C'_{12}\varepsilon_x + C'_{22}\varepsilon_y \tag{4}$$

$$\tau_{xy} = C'_{33}\varepsilon_{xy} \tag{5}$$

The values of the coefficients C'_{11} to C'_{33} for isotropic materials are given in Appendix- A.
The strain displacement relations for small displacements are:

$$\varepsilon_x = \alpha u \tag{6}$$

$$\varepsilon_y = \beta v \tag{7}$$

$$\varepsilon_{xy} = \alpha v + \beta u \tag{8}$$

Where $\alpha = \frac{\partial}{\partial x}, \beta = \frac{\partial}{\partial y}$

Eliminating $\sigma_x, \sigma_y, \tau_{xy}$ and all strain components between the above equations, the following equations are obtained, which can be written in matrix form as:

$$\frac{\partial}{\partial y}\begin{bmatrix} u \\ v \\ Y \\ X \end{bmatrix} = \begin{bmatrix} 0 & -\alpha & 0 & 1/G \\ C_1\alpha & 0 & C_2 & 0 \\ 0 & 0 & 0 & -\alpha \\ C_3\alpha^2 G & 0 & C_1\alpha & 0 \end{bmatrix}\begin{bmatrix} u \\ v \\ Y \\ X \end{bmatrix} \tag{9}$$

Where,

$X = \tau_{xy}$, $Y = \sigma_y = C'_{12}\varepsilon_x + C'_{22}\varepsilon_y$

$C_1 = \frac{-a_{12}}{a_{22}}; C_2 = \frac{1}{Ga_{22}}; C_3 = \frac{a_{12}}{a_{22}} - a_{11}$ and $a_{11} = \frac{C'_{11}}{G}, a_{12} = \frac{C'_{12}}{G}, a_{22} = \frac{C'_{22}}{G}$

eq. (9) can be expressed as:

$$\frac{\partial}{\partial y}\{S\} = [D]\{S\} \tag{10}$$

The solution of eq. (10) is

$$\{S\} = \left[e^{[D]y}\right]\{S_0\} \tag{11}$$

$$\{S\} = [L_i]\{S_0\} \tag{12}$$

Where $\{S_0\}$ is the vector of initial functions, being the value of the state vector {S} on the initial plane.
If u_0, v_0, Y_0 and X_0 are values of u, v, Y and X respectively, on the initial plane, then
$\{S_0\} = [u_0, \quad v_0, \quad Y_0, \quad X_0]^T$

Where $$[L_i] = e^{[D]y} \tag{13}$$

Expanding eq. (13) in the form of a series:

$$[L_i]=[I]+y[D]+\frac{y^2}{2!}[D]^2+....... \quad (14)$$

Where, [I] is a unit matrix.

Eq. (14) is suitably truncated depending on the order of the beam theory desired (Patel et al, 2014).The transfer matrix [L_i] relates the stresses and displacements at the bottom plane of the i-th layer to the same in any other parallel plane within the same layer (i= 1……n). Transfer matrix [L_i] is of 4x4, it can be generalized by [L_i]. The expressions for the terms in the matrix [L_i] are presented in Appendix- B.

ANALYSIS OF BEAMS

We have considered an isotropic beam of length l, depth d and loaded with a load W in the y-direction for the analysis purpose. Load W may be any one of uniformly distributed load, sinusoidal load, varying load or triangular load.

The bottom plane of the beam is taken as the initial plane. Due to loading at the top plane of the beam one has $X_0 = Y_0 = 0$.

On the plane, y = d, the conditions are X = 0, Y = -W.

After simplification yields the governing partial differential equation:

$$(L_{Yu}.L_{Xv}-L_{Yv}.L_{Xu})\phi=-W \quad (15)$$

Initial functions are obtained by substituting the value of Φ:

$\phi=A_0Sin\left(\frac{\pi x}{l}\right)$, The auxiliary function Φ is taken such that it satisfies the governing differential eq.(15) , as well as the boundary conditions at the edges of the beam. .

All the loads are taken in the form of sine function to satisfy boundary conditions and auxiliary function. From eq. (15) we get the value of A_0

$$u_0=L_{Xv}\phi,$$
$$v_0=-L_{Xu}\phi \quad (16)$$

From eq. (16), the values of initial functions are obtained (Patel et al, 2014).

The following values of beam dimensions are chosen:

b=200 mm, d=300 mm and l = 3000 mm

The following material properties are considered, E= 200000 N/mm^2, G = 76923 N/mm^2, μ = 0.3

The boundary conditions of the simply supported edges are:

X = Y = v = 0, at x = 0 and x = l

Considering different types of loading of intensity, W_0 = 100 N/mm in every case:

Since auxiliary function Φ is in terms of sine function. So, all the loads are converted in such a way that it contains the sine function using Fourier series taking only first term to satisfy the governing differential eq. (15).

(i) Uniformly distributed load $W(x)=\frac{4W_0}{\pi}Sin\frac{\pi x}{l}$

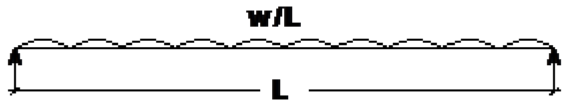

Figure 1 Simply supported beam carrying uniformly distributed load

(ii) Sinusoidal load $W(x) = W_0 Sin\left(\frac{\pi x}{l}\right)$ considering only one half sine wave.

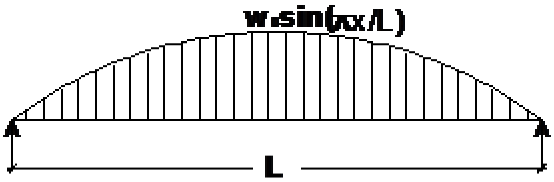

Figure 2 Simply supported beam carrying Sinusoidal load

(iii) Uniformly varying load $W(x) = \frac{2W_0}{\pi} Sin\left(\frac{\pi x}{l}\right)$

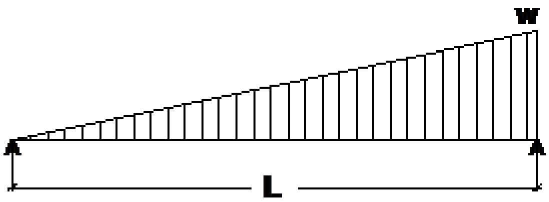

Figure 3 Simply supported beam carrying uniformly varying load

(iv) Triangular load $W(x) = \frac{8W_0}{\pi^2} Sin\left(\frac{\pi x}{l}\right)$

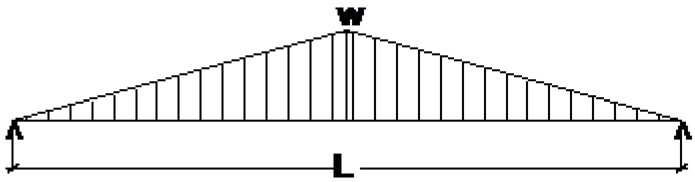

Figure 4 Simply supported beam carrying triangular load

The boundary conditions are exactly satisfied by the auxiliary function $\Phi = A_0 \sin(\pi x/l)$. Different types of loads of intensity W_0 = 100.0 N/mm is applied on the top surface of the beam. The value of auxiliary function (Φ) is obtained from eq. (15). Utilizing this value of auxiliary function the values of initial functions u0 and v0 are obtained from eq. (16). These are substituted in eq. (11) for obtaining the values of displacements and stresses.

RESULTS AND DISCUSSION

The values of displacements and stresses are obtained using method of initial functions. The results obtained by MIF are compared with bending theory. The displacements and stresses across the thickness in the particular problem of concrete beams are presented in Figures 5 to 9.

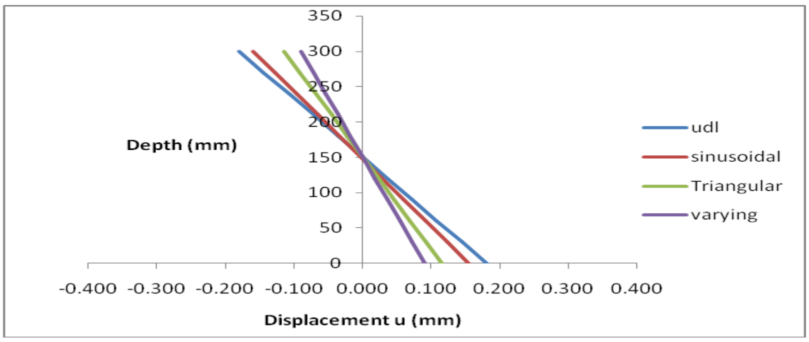

Figure 5 Variation of "Displacement u" for different loadings.

The displacement (u) is maximum in case of uniformly distributed load and mimimum in case of varying load. Its value is more at the top surface of the beam as compared to the bottom surface.

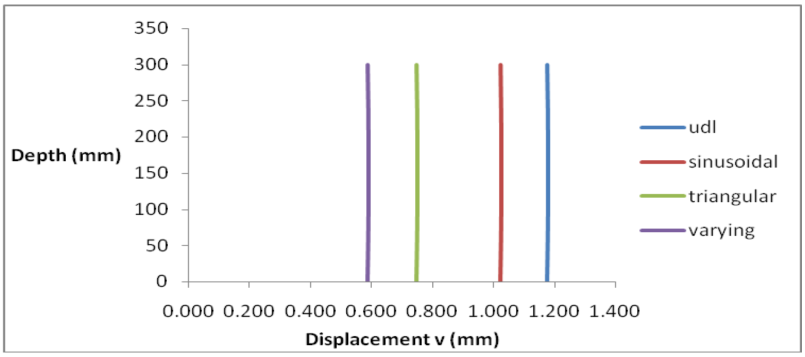

Figure 6 Variation of “Displacement v” for different loadings.

The variation of displacement (v) is almost linear across the depth. Its value is maximum i.e. 1.68 mm in case of uniformly distributed load and minimum i.e. 0.73 mm in case of varying load.

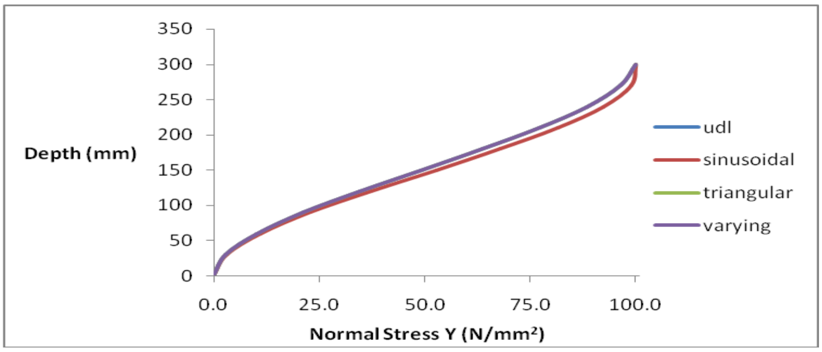

Figure 7 Variation of “Normal stress Y” for different loadings.

It is seen that the normal stress (Y) varies from zero at bottom surface of beam to maximum at top surface of beam.

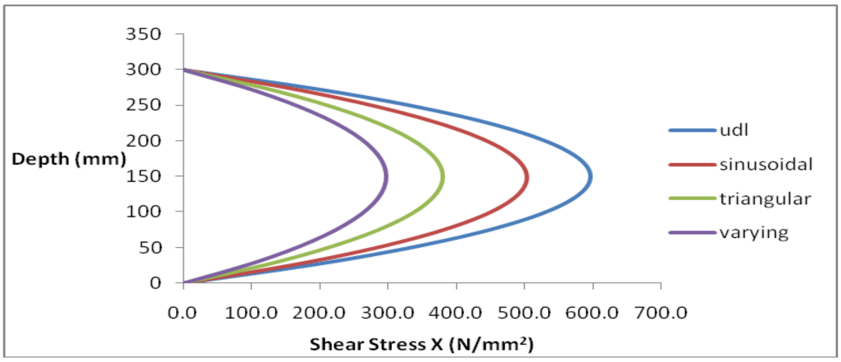

Figure 8 Variation of “Shear stress X” for different loadings.

From the figure it is observed that shear stress (X) is maximum in case of uniformly distributed load and minimum in case of varying load. Shear stress distribution is parabolic in all the four types of loading.

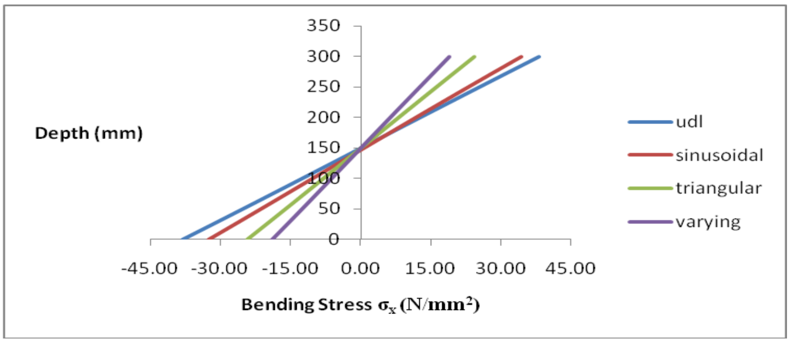

Figure 9 Variation of "Bending stress σ_x" for different loadings.

From the results is observed that the distribution of bending stress across the depth of beam section is linear in all the four cases. It is observed that bending stress is maximum in case of uniformly distributed load and minimum in case of varying load. Maximum value of bending stress is 37.02 N/mm^2 and minimum value is 18.51 N/mm^2.

CONCLUSIONS

Method of initial functions gives correct result for the different types of loadings. In this method assumptions are not taken regarding the behaviour of beams. It is observed that there is considerable effect of different types of loadings on the behaviour of beams. It is observed the deflection and stresses are different for different types of loadings despite of same intensity. For uniformly distributed load, the maximum value of deflection is 1.19 mm and the maximum value of bending stress is 37.02 N/mm^2.

Minimum values of deflection and stresses are observed in case of varying load, which are 0.59 mm and 18.51 N/mm^2 respectively. Value of normal stress is equal to the intensity of loading in all cases; it confirms that MIF is successfully applied for different types of loadings.

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